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In the realm of successive sampling, most of the literature concerns with the estimation is population mean and no emphasis is laid on estimation of population variance. Motivated by Isaki's (1983) work of variance estimation, Singh et al. (2011) put their first effort on estimation of population variance under successive sampling. Thus, by cognizing aforementioned problem, we proposed combined estimators for estimating population variance precisely and an analytical scenario is also presented for judging its properties. A numerical illustration, which validate the usefulness of the proposed estimator, based on hypothetical population is also mentioned.

Keywords: Successive sampling, mean square error and optimum replacement policy.

1 Introduction

The technique of successive sampling is prevalent in a wide variety of contexts due to a realization that with a dynamic population a census at infrequent intervals is of limited use. To broader the horizons of sampling techniques, Jessen (1942) in statistical investigation of sample survey for obtaining farm facts investigated "matching" as a special case of double smpling and introduced new sampling technique as successive sampling. He utilized the information obtained on earlier occassion with the partial replacement of sampling unit for improving the estimates of mean of the current occasion. After Jessen, Patterson (1950) confronting the same sampling technique with the partial replacement of units and provide current estimates and estimte of change. Further, the caliber of this sampling procedure was recognized and extended by Eckler (1955), Rao and Graham (1964), Singh and Singh (1965), Sen (1971,72,73a,73b), Kathuria (1975), Tikkiwal (1951, 53,56,58,60,64,65,67), Raj (1965), Singh (1968), Singh and Kathuria (1969), Ghangurde and Rao (1969), Avadhani and Srivastava (1972), Chotai (1974), Arnab (1979), Sen et al.(1975), Chaudhuri and Arnab (1977,79), Adhvaryu (1978), Gupta (1979), Tripathi and Srivastava (1979), Singh (1980), Kumar and Gupta (1981), Das (1982), Chaturvedi and Tripathi (1983), Okafor (1987,92), Chaudhari and Graham (1983), Srivastava and Jhajj (1987), Tripathi et al. (1989), Okafor and Arnab (1987), Arnab and Okafor (1992), Singh et al. (1992), Singh and Yadav (1992), Prasad and Graham (1994), Birdar and Singh (2001) etc.

With a general approach and sampling strategy Singh et al (2011) proposed estimators for estimation of population variance in successive sampling. Ahmed et al (2016), Singh and Singh (2016) extended the same sampling strategy and deviced estimators. On the similar lines, we proposed some estimators with efficient sampling strategy in subsection (2.2). An optimum replacement policy, expressions for efficiency and percentage gain in precision, and expressions for cost efficiency and percentage gain in precision are checked out in section (4).

2 Sampling Methodology

2.1 Notations

Let $U = U_1, U_2, ..., U_N$ be a finite population of size N, which has been sampled over two occasions. Let the character understudy be denoted by x and y on the first and second occasion respectively. Let the sizes of both the samples drawn using simple random sampling without replacement on both the occasion be n. We use two phase sampling in the matched proportion for which large sample is the first sample of size n for study variable x and small sample is of size m units for the study variable y at the second occasion.
In successive sampling in selecting the second smaple of size n, m of the units in the first sample are retained. The rest $u(= n-m)$ units are chosen as the new units selected independently of the matched portion. The current paper adopts the information obtained from the first occasion to estimate the population variance on the second occasion. Let $(x_1, x_2, ..., x_n)$ be the $'n'$ units of study variable x drawn by SRSWOR at occasion first. From these obtained n units of first occasion, we draw m units such as $(y_1, y_2, ..., y_m)$ of the study variable as matched units on the second occasion; $(y_1^*, y_2^*, ..., y_u^*)$ be the values of the study variable y for the unmatched portion on the second occasion. The following notations are envisaged for the further use

$\bar{X}$ and $\bar{Y}$ are the population means of the study variables at occasion first and second respectively.
S_x^2,and S_y^2 are the sampling variance of the variables written as in subscript.
s_{xn}^2,s_{xm}^2, s_{ym}^2 and s_{yu}^2 are the sample variance of the variables written as in subscript with their respective sizes.

C_x and C_y be the coffecient of variation of the study variables at first and second occasion respectively.

$Q = \dfrac{u}{n}$ fraction of unmatched sample

$P = \dfrac{m}{n}$ fraction of matched sample

2.2 Proposed Estimator

Motivated by Singh et al (2011), we present some combined estimator $T_i^{'}$ of finite population variance based on matched and unmatched proportions are

$$T_i^{'} = \phi_i T_{mi} + (1-\phi_i)T_u; \quad i = 1,2,3,4,5 \tag{2.1}$$

here, $T_{m1} = s_{ym}^2 + \beta^{'}(s_{ym}^2 - s_{xn}^2)$, $T_{m2} = s_{ym}^2\left[1+log\left(\frac{s_{xm}^2}{s_{xn}^2}\right)\right]^{\beta_0}$, $T_{m3} = s_{ym}^2\left[1+\beta_1 log\left(\frac{s_{xm}^2}{s_{xn}^2}\right)\right]$, $T_{m4} = s_{ym}^2\left[1+log\left(\frac{s_{xm}^{*2}}{s_{xn}^{*2}}\right)\right]^{\beta_2}$, $T_{m5} = s_{ym}^2\left[1+\beta_3 log\left(\frac{s_{xm}^{*2}}{s_{xn}^{*2}}\right)\right]$ and $T_u = s_{yu}^2$

where, $s_{xn}^{*2} = as_{xn}^2 + b$ and $s_{xm}^{*2} = as_{xm}^2 + b$ such that $a(\neq 0)$ and b are either the real numbers or functions of the known population parameters at the first occasion such as the standard deviations, coefficient of kurtosis, coefficient of variation, and correlation coefficient of the population.

The mean square error of the estimator is derived up to the first order of approximations under large sample assumptions and using the following transformations:
$s_{ym}^2 = S_y^2(1+\epsilon_0)$
$s_{xm}^2 = S_x^2(1+\epsilon_1)$
$s_{xn}^2 = S_x^2(1+\epsilon_2)$
and $s_{yn}^2 = S_y^2(1+\epsilon_3)$ such that $E(\epsilon_i) = 0;\ \forall i = 0,1,2,3$
$E(\epsilon_0^2) = \frac{1}{m}(\lambda_{40}-1) = \frac{1}{m}\lambda_{40}^*$
$E(\epsilon_1^2) = \frac{1}{m}(\lambda_{04}-1) = \frac{1}{m}\lambda_{04}^*$
$E(\epsilon_2^2) = \frac{1}{n}(\lambda_{04}-1) = \frac{1}{n}\lambda_{04}^*$
$E(\epsilon_0\epsilon_1) = \frac{1}{m}(\lambda_{22}-1) = \frac{1}{m}\lambda_{22}^*$
$E(\epsilon_0\epsilon_2) = \frac{1}{m}(\lambda_{22}-1) = \frac{1}{n}\lambda_{22}^*$
where $\lambda_{rs} = \frac{\mu_{rs}}{\sqrt{\mu_{20}^r\mu_{02}^s}}$ and $\lambda_{rs}^* = \lambda_{rs} - 1 \quad \forall r,s = 0,1,2,3,4$

3 MSE of the Proposed Estimator

Theorem 1. *The mean square error of the proposed estimator $T_1^{'}$ to the first order of approximation is given by*

$$M(T_1^{'}) = \frac{1}{n}\frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}S_y^4\lambda_{40}^* \tag{3.1}$$

where, $\rho^* = \frac{\lambda_{22}^*}{\sqrt{\lambda_{40}^*}\sqrt{\lambda_{04}^*}}$

Proof. The MSE of the proposed estimator $T_1^{'}$ is given by

$$M\Big(T_1^{'}\Big) = \Big(1-\phi_1\Big)^2 M\Big(T_u\Big) + \phi_1^2 M\Big(T_{m1}\Big) \tag{3.2}$$

For the minimum variance of the estimator $T_1^{'}$, we differentiate above expression with respect to ϕ_1, we get

$$\phi_{opt} = \frac{M\Big(T_u\Big)}{M\Big(T_u\Big) + M\Big(T_{m1}\Big)} \tag{3.3}$$

Now, MSE of the unmatched portion and matched portion of the suggested combined estimator is given by

$$M\Big(T_{u1}\Big) = \frac{S_y^4}{u}\lambda_{40}^* \tag{3.4}$$

and

$$\begin{aligned} T_{m1} &= s_{ym}^2 + \beta^{'}(s_{xm}^2 - s_{xn}^2) \\ &= S_{ym}^2(1+\epsilon_0) + \beta^{'}\left[S_{xm}^2(1+\epsilon_1) - S_{xn}^2(1+\epsilon_2)\right] \end{aligned}$$

$$\big(T_{m1} - S_y^2\big) = \left[S_y^2\epsilon_0 + \beta^{'} S_y^2\big(\epsilon_1 - \epsilon_2\big)\right]$$

On squaring both sides and then taking expectation on both sides of the above expression1, we get

$$E\big(T_{m1} - S_y^2\big)^2 = E\big[S_y^2\epsilon_0 + \beta^{'} S_y^2\big(\epsilon_1-\epsilon_2\big)\big]^2$$

$$\begin{aligned} M\big(T_{m1}\big) &= E\big[S_y^4\epsilon_0^2 + \beta^{'2}S_x^4\big(\epsilon_1-\epsilon_2\big)^2 + 2\beta^{'}S_y^2S_x^2\big(\epsilon_0\epsilon_1 - \epsilon_0\epsilon_2\big)\big] \\ &= \frac{\lambda_{40}^*}{m}S_y^4 + \beta^{'2}\Big(\frac{1}{m}-\frac{1}{n}\Big)\lambda_{04}^*S_x^4 + 2\Big(\frac{1}{m}-\frac{1}{n}\Big)\beta^{'}\lambda_{22}^*S_y^2S_x^2 \\ &= \frac{\lambda_{40}^*}{m}S_y^4 + \Big(\frac{1}{m}-\frac{1}{n}\Big)\Big[\beta^{'2}\lambda_{04}^*S_x^4 + 2\beta^{'}\lambda_{22}^*S_y^2S_x^2\Big] \end{aligned}$$

For minimum variance of the proposed estimator, differeniate above expression with respect to β, we get

$$\begin{aligned} \frac{\partial M\big(T_{m1}\big)}{\partial\beta^{'}} &= 0 \Rightarrow 2\beta^{'}S_x^4\beta_{2x}^* + 2S_y^2S_x^2\lambda_{22}^* = 0 \\ &\Rightarrow \beta^{'} = \frac{-S_y^2\lambda_{22}^*}{S_x^2\beta_{2x}^*} \end{aligned}$$

Thus, we have minimum variance as follows

$$\begin{aligned} M^*\big(T_{m1}\big) &= \frac{\lambda_{40}^*}{m}S_y^4 - \Big(\frac{1}{m}-\frac{1}{n}\Big)\frac{\lambda_{22}^*}{\lambda_{04}^*}S_y^4 \\ &= \frac{\lambda_{40}^*}{m}S_y^4\Big(1 - \frac{\lambda_{22}^{*2}}{\lambda_{40}^*\lambda_{04}^*}\Big) + \frac{S_y^4}{n}\frac{\lambda_{22}^{*2}}{\lambda_{04}^*} \\ &= \frac{\lambda_{40}^*}{m}S_y^4\Big(1-\rho^{*2}\Big) + \lambda_{40}^*\rho^{*2}\frac{S_y^4}{n} \end{aligned}$$

$$= S_y^4 \lambda_{40}^* \Big[\frac{1-\rho^*2}{m} + \frac{\rho^*2}{n}\Big]$$

$$= S_y^4 \lambda_{40}^* \frac{1}{k_A} \tag{3.5}$$

where $\frac{1}{k_A} = \Big[\frac{1-\rho^*2}{m} + \frac{\rho^*2}{n}\Big]$ and $\rho^{*2} = \frac{\lambda_{22}^{*2}}{\lambda_{40}^* \lambda_{04}^*}$

So, expreesion ϕ_{1opt} reduces as follows

$$\phi_{1opt} = \frac{\frac{S_y^4}{u}\lambda_{40}^*}{\Big[\frac{S_y^4}{k_A} + \frac{S_y^4}{u}\Big]\lambda_{40}^*} = \frac{k_A}{k_A + u} \tag{3.6}$$

$$1 - \phi_{1opt} = \frac{u}{k_A + u} \tag{3.7}$$

Now, the MSE of the combined estimator, by using the expressions (3.2), (3.4), (3.5), (3.6) and (3.7), we get

$$M\Big(T_1^{'}\Big) = \Big(1 - \phi_1\Big)^2 M\Big(T_u\Big) + \phi_1^2 M\Big(T_{m1}\Big)$$

$$= \frac{u^2}{k_A + u}\frac{S_y^2}{u}\lambda_{40}^* + \frac{k_A^2}{k_A + u}\frac{S_y^4}{k_A}\lambda_{40}^*$$

$$= \frac{S_y^4}{k_A + u}\lambda_{40}^*$$

$$= \frac{1}{\left[\dfrac{1-\rho^*2}{m} + \dfrac{\rho^*2}{n}\right]^{-1} + u} S_y^4 \lambda_{40}^*$$

$$= \frac{1}{\left[\dfrac{n - n\rho^{*2} + m\rho^{*2}}{mn}\right] + u} S_y^4 \lambda_{40}^*$$

$$= \frac{1}{\left[\dfrac{n - u\rho^{*2}}{mn}\right] + u} S_y^4 \lambda_{40}^*$$

$$= \frac{n - u\rho^{*2}}{n^2 - u^2\rho^{*2}} S_y^4 \lambda_{40}^*$$

$$= \frac{1}{n}\frac{1 - Q\rho^{*2}}{1 - Q^2\rho^{*2}} S_y^4 \lambda_{40}^* \tag{3.8}$$

where $Q = \frac{u}{n}$ and $P = 1 - Q = \frac{m}{n}$ are unmatched and matched portion respectively.

□

Theorem 2. *The mean square error of the proposed estimator $T_i^{'}$ to the first order of approximation is*

$$M(T_j^{'}) = \frac{1}{n}\frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}S_y^4\lambda_{40}^{*}; \quad j = 2,3,4,5 \tag{3.9}$$

where, $\rho^{*} = \dfrac{\lambda_{22}^{*}}{\sqrt{\lambda_{40}^{*}}\sqrt{\lambda_{04}^{*}}}$

Proof. We can proof the theorem 2 on the similar pattern of the theorem 1. □

4 Analytical Study

Optimization is paramount to any problem involving decision making in any scientific domain. We also put here an analysis regarding our proosed strategy, namely optimum replcement policy. Further, gain in precision is considered by ignoring the cost of sampling operations.

4.1 Optimum Replacement Policy

Under replacement policy, we optimize values of unmatched proportion Q (*or* u) and matched proportion P (*or* m) for the proposed estimators are obtained by

$$\frac{\partial T_i^{'}}{\partial Q} = 0; \qquad i = 1,2,3,4,5$$

We get same quadratic expression by minimizing above expression from all the proposed estimators as follows

$$\rho^{*2}Q^2 - 2Q + 1 = 0$$

$$Q^{*} = \frac{2 \pm \sqrt{4-4\rho^{*2}}}{2\rho^{*2}}$$

$$Q^{*} = \left(1+\sqrt{1-\rho^{*2}}\right)^{-1}$$

Hence, we get

$$P^{*} = \frac{\sqrt{1-\rho^{*2}}}{1+\sqrt{1-\rho^{*2}}} \tag{4.1}$$

Thus, by putting optimum values of P and Q, we obtain optimum mean square error as follows

$$M_{opt}\left(T_i^{'}\right) = \frac{1-\left(1+\sqrt{1-\rho^{*2}}\right)^{-1}\rho^{*2}}{1-\left(1+\sqrt{1-\rho^{*2}}\right)^{-2}\rho^{*2}}\frac{S_y^4}{n}\lambda_{40}^{*}$$

$$= \frac{S_y^4}{2n}\,\lambda_{40}^{*}\left(1+\sqrt{1-\rho^{*2}}\right)$$

4.2 Gain in Precision

Ignoring the cost of the sampling operations the percentage proportional gain due to matching over no matching for S_y^2 is given by

$$Gain = \left[\frac{\frac{1}{n}S_y^4\lambda_{40}^* - \frac{1}{n}\left(\frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}\right)S_y^4\lambda_{40}^*}{\frac{1}{n}\left(\frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}\right)S_y^4\lambda_{40}^*}\right]100\%$$

$$= \left[\frac{1-Q^2\rho^{*2}}{1-Q\rho^{*2}} - 1\right]100\% \tag{4.2}$$

Further, if we wish to use matched samples the propotional increase in the variance resulting from the deviation from optimum matching for S_y^2 is given by

$$prop.inc.M(T_i^{'}) = \left[\frac{M(T_i^{'})}{M_{opt}(T_i^{'})} - 1\right]100\%$$

$$= \left[\frac{2(1-Q\rho^{*2})}{(1-Q^2\rho^{*2})(1+\sqrt{1-\rho^{*2}})} - 1\right]100\% \tag{4.3}$$

4.3 Cost Efficiency and Gain in Precision

Following Kulldorf (1963), we also consider the case of cost by taking the total cost apart from fixed cost on the sacond occasion. Let the total cost except the fixed cost on the second occasion is given by

$$C_2 = mc_m + uc_u \tag{4.4}$$

where c_m = per unit cost for matched portion
and c_u = per unit cost for unmatched portion

Dividing by c_u , we have

$$\frac{C_2}{c_u} = m\delta + u$$

$$\frac{C_2}{c_u} = \delta n + \Big(1-\delta\Big)u$$

where, $\delta = \frac{c_m}{c_u}$.

So, on the second occasion, the optimum unmatched proportion $Q = \frac{u}{n}$ for proposed sampling strategies with the above cost structure can be obtained by minimizing the following function

$$\frac{MC_2}{c_uS_y^4} = \frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}\Big[\delta + \Big(1-\delta\Big)Q\Big]\lambda_{40}^*$$

with rescpect to Q

The optimum value of Q minimizing the above expression is given by

$$\frac{\partial}{\partial Q}\left(\frac{MC_2}{c_u S_y^4}\right)=0$$

$$\Big(1-\delta-\rho^{*2}\delta\Big)\rho^{*2}Q^2+2\Big(2\delta-1\Big)Q+\Big(1-\delta-\rho^2\delta\Big)=0$$

$$Q^*_{(opt)}=\frac{-\Big(2\delta-1\Big)\pm\sqrt{\Big(2\delta-1\Big)^2-\big(\frac{(1-\delta-\rho^{*2}\delta)^2}{\rho^{*2}}\big)}}{\Big(1-\delta-\rho^{*2}\delta\Big)}$$

When the costs of sampling operations are considered the percentage proportional gain due to matching over no matching for the proposed sampling strategy is given by

$$Gain_{c,nomatch}=\left[\frac{\Big(\frac{1}{n}S_y^4\lambda_{40}^*\Big)nc_u}{\frac{1}{n}\left(\frac{1-Q^*_{(opt)}\rho^{*2}}{1-Q^{*2}_{(opt)}\rho^{*2}}\right)\left(\delta+(1-\delta)Q^*_{(opt)}\right)S_y^4\lambda_{40}^*}-1\right]100\%$$

$$=\left[\frac{\Big(1-Q^{*2}_{(opt)}\rho^{*2}\Big)}{\Big(1-Q^*{}_{(opt)}\rho^{*2}\Big)\left(\delta+(1-\delta)Q^*_{(opt)}\right)}-1\right]100\% \qquad (4.5)$$

Further under the optimum matching the percentage proportional increase in the product of variance and the corresponding per unit cost due to deviation from the optimum matching for the proposed sampling strategy is given by

$$prop.inc.M(T_i^{'})=\left[\frac{M(T_i^{'})C_2}{M_{opt}(T_i^{'})C_{2(opt)}}-1\right]100\%$$

5 Numerical Illustration

Table 1: Optimum Matched Percentage and Percentage Gain in Precesion

ρ^*	$\left(\frac{m}{n}\right)_{opt}$	% gain in precision for $\left(\frac{m}{n}\right)_{opt}$	% gain in precision for $\left(\frac{m}{n}\right)=\frac{1}{2}$	% gain in precision for $\left(\frac{m}{n}\right)=\frac{1}{3}$	% gain in precision for $\left(\frac{m}{n}\right)=\frac{1}{4}$
0.6.	44	11.1111	10.97.561	10.526316	9.246575
0.7.	41	16.61639	16.622517	16.17162	14.2569
0.8.	37	25	23.52941	24.80620	23.07692
0.9.	30	39.28645	34.0361	39.13043	38.69427
0.95.	24	52.40999	41.11617	50.34868	52.36944
1.0.	0	–	50.0000	66.666	75

Table 2: Proportional increase in variance when the proposed sampling strategy is used

ρ^*	propotional increase in variance for $\left(\frac{m}{n}\right)=\frac{1}{2}$	propotional increase in variance for $\left(\frac{m}{n}\right)=\frac{1}{3}$	propotional increase in variance for $\left(\frac{m}{n}\right)=\frac{1}{4}$
0.6.	0.1221001	0.5291005	1.7067224
0.7.	0.3882336	0.4345067	1.8779183
0.8.	1.1904762	0.1552795	1.5625000
0.9.	3.9190411	0.1121329	0.4269667
0.95.	8.0032084	1.3710260	0.0266165
1.0.	33.3333333	20.0000000	14.2857143

6 Interpretation and Conclusion

The table (1) give the results of the optimum matched percentage of the sample obtained by (4.1) for the proposed successive sampling strategy and compared with no matching, the percentage gain in precision is given by (4.5) for given ρ^*. The best percentage to match never exceeds 50% and decreases rapidily as we increase ρ^*. When $\rho^* = 1$, then we have matched proportion 0. We can also explain it as, as sample size or population size become large, then we can not be improve the estimator of variance at the estimation stage. Further the percentage gain due to matching with $\frac{m}{n} = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$ compared with no matching is tabulated in table (2). As matching proportion is decreses, we can see from table that percentage gain in precision is increased as we increase ρ^*. Certainly, the whole study improve the estimation procedure and lengthen the idea in the realm of the successive sampling and wipe out the existing gap.

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