EasyChair Preprint

№ 187

On the convergence of driver centric zone pricing for traffic networks

Péter Kovács, Balázs Ádám Kulcsár, Henk Wymeersch and András Bálint

EasyChair preprints are intended for rapid dissemination of research results and are integrated with the rest of EasyChair.

May 30, 2018

On the convergence of driver centric zone pricing for traffic networks

Péter Kovács * **Balázs Ádám Kulcsár** * **Henk Wymeersch** **
András Bálint ***

* *Automatic Control Group, Department of Electrical Engineering, Chalmers University of Technology, Gothenburg, Sweden (e-mail: peterko@chalmers.se, kulcsar@chalmers.se).*
** *Communication Systems Group, Department of Electrical Engineering, Chalmers University of Technology, Gothenburg, Sweden, (e-mail: henkw@chalmers.se).*
*** *Division of Vehicle Safety, Department of Mechanics and Maritime Sciences, Chalmers University of Technology, Gothenburg, Sweden, (e-mail: andras.balint@chalmers.se).*

Abstract: Congestion fees with dynamical zone pricing provide an easy-to-implement solution to urban traffic control agencies for improved efficiency of the traffic network. In order to design a pricing strategy it is necessary to understand how it affects the route choices of drivers in the system. This work explores this question by providing an idealized dynamical model for re-routing as traffic flows change to cheaper routes. Results show a convergence to a Wardrop-equilibrium, which is also proven formally and demonstrated via simulation.

Keywords: Urban traffic control, Zone pricing, Route choice, User equilibrium

1. INTRODUCTION

One powerful possibility to influence traffic is the introduction of congestion fees. Throughout this work we examine how drivers react to being charged for entering certain urban areas. We assume traffic flows adhere to *Wardrop's principles*, namely that

- **more costly routes carry no flow**, referred to as the user equilibrium (UE) principle and
- **the average route cost is minimal**, referred to as system equilibrium (SE).

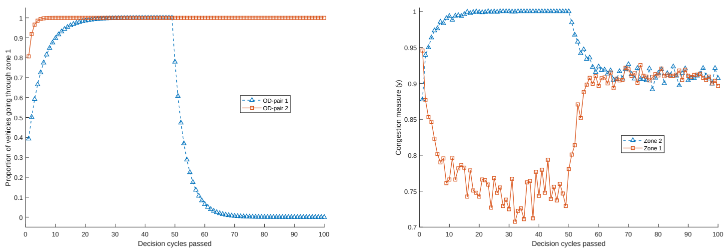
As we wish to build a two-commodity (route choice, congestion fee) dynamical system in order to design a control algorithm for pricing, we first need to understand how the user equilibrium is reached. Our work covers this question in detail.

When modeling the route choices of drivers we adapt the *restricted proportional adjustment process* (RPAP). This was introduced by Liu and Smith (2015) to design signal timings optimizing route-choice on a link-based level; in contrast we aim to influence driver behavior by a zone-based congestion pricing system. We propose to apply pricing on a zone-based level for various reasons. From a practical standpoint it is easier to implement and realize as it requires less computational power to form congestion level predictions and also less infrastructural investment. On the other hand, from a theoretical aspect since the pricing is aimed at controlling a larger geographical area, and thus is applied at a different hierarchical level the models for congestion require less accuracy. For congestion modeling we apply techniques from queueing theory (for a detailed discussion see Adan and Resing (2002)) and traffic flow theory, more precisely the *macroscopic fundamental diagram* (MFD) as discussed by Geroliminis and Daganzo (2008).

2. MODEL

In our work we use a simplified model of an urban traffic network with the following criteria. We assume the city where the pricing control would be put into use can be divided into areas or zones in such a way, that the congestion patterns and general behavior of traffic within an area is similar, thus we can model the aggregate behavior and the resulting congestion level there by a queue. These queues do not necessarily represent actual queues of traffic, they are rather representations of the traffic conditions which locally determine the delays in travel time exhibited by the drivers. Thus control is applied on these zones as well, dynamically assigning prices to them, which drivers will have to pay upon entering them. The total price a vehicle has to pay will be calculated as the sum of the prices of the zones it enters on its route.

Routes are modelled as a set of links (with a standard directed graph setting of traffic links and junctions), connecting a link where the vehicle exogenously enters the network (origin) and where it finally leaves the network (destination). We call a compact set of links a zone, and we assume that within a zone a route always follows the shortest path. Zones are indexed by i, OD-pairs are indexed by j. Thus we can associate every route r with the OD-pair it connects ($r \in \mathcal{R}_j$) and the set of zones ($\mathcal{I}_r$) it passes through. The route choice happens on the finite



set of routes each OD-pair has ($\mathcal{R}_j$). Furthermore we call a set of links which are connected in a directed order that contains no loops a *segment*. Every route is assumed to be a segment as well. We use the notation "$\sim$" for a relation between two routes (r and s) to signal that

- they belong to the same OD-pair,
- the difference set $r - s$ is a segment,
- the difference set $s - r$ is a segment

which adopts the piecewise alternative segment (PAS) concept of Bar-Gera (2010). We only allow drivers to switch from route r to route s as long as $r \sim s$.

The drivers in the system can be of two types, *controlled*, who are subscribed to the incentive system and take its prices into account when making their decisions (denoted by c); and textitnon-controlled, who make routing decisions according to an independent discrete probability distribution (denoted by n). Both types arrive on their routes following a Poisson process with rate λ_r^c and λ_r^n, thus the arrivals on OD-pairs are also Poisson with rate $\lambda_j = \sum_{r \in \mathcal{R}_j} \sum_{t \in \{c,n\}} \lambda_r^t$. In stationarity it also holds that the arrivals to each zone is Poisson with rate $\lambda_i = \sum_{r: i \in \mathcal{I}_r} \sum_{t \in \{c,n\}} \lambda_r^t$. Departures from the queues happen following an on-off process, where the theoretical maximal rate of vehicles leaving a zone (i.e. the aggregate outflow capacity) is given by μ_i, and the process is on for y_i proportion of the time. Congestion is modeled through this y_i parameter, which is dependent on the demand on the network, i.e. $y_i = y_i(\boldsymbol{\lambda})$. With these assumptions in stationarity the queues work the same way as $M/G/1$ type queues, thus there is a closed form analytical expression for the average delay suffered by a driver passing through a zone, which we denote by $D_i = D_i(\boldsymbol{\lambda})$.

The controlled drivers' decisions are given by their utility functions, which for each route can be decomposed to zone-related terms as

$$U_r = \sum_{i \in \mathcal{I}_r} \left(-w_D(D_i + \tau_i r) - w_\gamma \gamma_i\right), \tag{1}$$

where $\tau_i r$ is the free-flow driving time through zone i on route r, γ_i is the price charged in zone i and w_D and w_γ are fixed weights representing the importance of these factors to the drivers. The changes in route choice are given through the arrival rates to each route. Namely, these change according to a discrete time process as described by the following differentia equations,

$$\begin{aligned} \Delta\lambda_r^c &= -\kappa\lambda_r^c (U_s - U_r)^+ + \kappa\lambda_s^c (U_r - U_s)^+ \\ \Delta\lambda_s^c &= +\kappa\lambda_r^c (U_s - U_r)^+ - \kappa\lambda_s^c (U_r - U_s)^+, \end{aligned} \tag{2}$$

where r and s are routes such that, $r \sim s$; $x^+ = x$ if $x > 0$ and $x^+ = 0$ otherwise; and κ is a fixed a parameter of the model. We point out, that, though not explicitely stated $\mathbf{U} = \mathbf{U}(\boldsymbol{\lambda})$, due to the delays' dependence on the arrival rates. Our results concern the stability of the dynamical system driven by this process.

3. RESULTS

The results of our work show that under fixed prcing the system desribed above converges to a user equilibrium (UE), which satisfies Wardrop's first condition as routes with lower utility carry no flow, while routes carrying flow possess equal utilities. In our theoretical work, we rigorously formulate these UEs and provide a proof of convergence using a Lyapunov argument. This result shows the viability of the concept of incentivizing drivers with dynamically controlled congestion fees, as the system reaches approximate equilibria within finite time steps, thus if the frequency of price changes is properly chosen, then the system can be expected to perform close to system equilibrium.

We also provide a simulation study to demonstrate convergence to the user equilibrium. We examined a simple network consisting of two OD-pairs with two routes each going through one of two zones. We chose demands, which produce a visible drop in capacity due to congestion, yet were far enough from the boundary of the capacity region that even with all controlled vehicles choosing the same route, the system remained stable. After half of the runtime we changed the pricing to represent a fairer pricing. Figure 1 shows the results in terms of proportions of vehicles choosing a route through zone 1 (left), and congestions levels in the zones (right). Our simulations results show a fast convergence to user equilibrium, which further emphasizes the applicability of such a pricing system.

REFERENCES

Adan, I. and Resing, J. (2002). *Queueing theory*. University of Technology Eindhoven.

Bar-Gera, H. (2010). Traffic assignment by paired alternative segments. *Transportation Research Part B: Methodological*, 44(8-9), 1022–1046.

Geroliminis, N. and Daganzo, C.F. (2008). Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B: Methodological*, 42(9), 759–770.

Liu, R. and Smith, M. (2015). Route choice and traffic signal control: A study of the stability and instability of a new dynamical model of route choice and traffic signal control. *Transportation Research Part B: Methodological*, 77, 123–145.