Representation Of Fuzzy Queries In The Form Of A Multi-Criteria Decision Matrix Using Z-Numbers

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Abstract. This work is devoted to the formalization of fuzzy conditions of fuzzy queries in the form of a Z-number without the direct intervention of experts. As you know, the number of alternatives involved in the decision-making process in multicriteria problems is usually not very large and is evaluated according to criteria with the help of experts. However, fuzzy queries are mainly used to evaluate and rank a large number of alternatives, for which it is almost impossible to carry out a preliminary assessment by criteria. On the other hand, how can you be sure of the reliability of the obtained results of fuzzy queries with fuzzy conditions, also expressed with certain reliability.

A method is proposed for converting fuzzy conditions of fuzzy queries (criteria) to Z-number, with the help of which it is possible to use multi-criteria decision-making methods of Z-numbers to process fuzzy queries with a large number of records (alternatives).

Keywords: Corporate information systems; Fuzzy queries; Fuzzy associations, Z-number, Decision-making methods, Decision-making matrix

1 Introduction

The data processed in the databases of modern corporate information systems, to which the wholesale and retail trade enterprises can be attributed, are of a clear, numerical nature [1]. However, queries to these databases are often vague. That is connected with the semantic ambiguity of the language, in particular, with the inherent vagueness of the textual information with the help of which queries to databases are formulated. In this regard, the concept of fuzzy queries to databases appeared, which is a promising direction for storing and processing information in modern corporate information systems..

A fuzzy query is a kind of flexible query , in which the user describes the requirements with fuzzy query conditions [1,4,5,6,7,9].

Mechanisms of fuzzy queries to relational databases based on the theory of fuzzy sets Zadeh [2], were first proposed in 1984 and subsequently developed in the works of D. Dubois and G. Prada [3,4] and other researchers [11,12,13, 14,15,16].

According to many authors [11,12,13], there are two main directions of research in the field of using fuzzy set theory in the context of a DBMS. The first assumes the use of a regular database and, in fact, develops a fuzzy query interface using fuzzy sets, possibility theory, fuzzy logic, etc. The second line of research uses fuzzy or probabilistic elements to develop a fuzzy database model.

In our research, we adhere to the concept: a fuzzy query system is an interface for users that allows you to obtain information from a database using a (quasi) sentence in a natural language [7]. Considering that there are some differences depending on the peculiarities of different implementations of the algorithm for processing fuzzy queries, however, the response to a fuzzy query sentence, as a rule, is a list of records ranked according to the degree of correspondence to the query [2].

Each of the fuzzy queries incorporates systems that can be considered as a multi-criteria decision-making problem for the solution of which there are many methods [8,17,19,20]. As you know, the alternatives involved in the decision-making process are usually not very large and are evaluated according to criteria with the help of experts. However, fuzzy queries are mainly used to evaluate and rank a large number of alternatives, for which it is almost impossible to conduct a preliminary assessment by experts on all criteria. On the other hand, how can experts practically assess the reliability of each fuzzy condition for each alternative? Recently, to solve the problem of evaluating the reliability of numbers, the concept of Z-number = number + its reliability has been used, which was first put forward by Zadeh. A number of decision-making methods have been developed based on the Z-number [2,]. In this paper, a method is proposed for converting the conditions of fuzzy queries to Z-number, with the help of which decision-making methods can be used to process fuzzy queries with a large number of alternatives.

The work is structured as follows. In Section 2, we present the necessary definitions and some material serving as a prerequisite. Section 3 formulates the problem statement. Section 4 presents a method and an algorithm for its solution. Section 5 shows an example of converting a fuzzy condition to the form Z-number. Final comments are included in section 6..

2 Basic Concepts and Definitions

This article discusses the issues of formulating fuzzy z-numbers for modeling linguistic uncertainty for processing fuzzy-flexible queries to a relational database using multi-criteria decision-making methods.

Definition 1. A fuzzy set [Zadeh,Aliyev] A is defined on a universe X may be given as:

$$A = \{\langle x, A(x)\rangle \mid x \in X\} \tag{1}$$

where $\mu_A : X \rightarrow [0; 1]$ is the membership function A. The membership value $\mu_A(x)$ describes the degree of belongingness of $x \in X$ in A.

Definition 2. Fuzzy queries. Fuzzy inquiries can be formalized as follows:

Select
<Attributes>,
<belonging functions of fuzzy concepts>,
<query membership indices>
From < A set of fuzzy A > join < fuzzy B set >
Where **<** conditions described by fuzzy concepts (LV) > ⊕ ...

< exact interval conditions > (2)

, join – shows the rules of merging on sets.

Thus, the terms of the query are as accurate interval terms as in form (2) (assortment number = [50,100], 5 months before expiration date (expiration date <5), etc.) and also as fuzzy terms (e.g. assortment number). goods with less, goods with good sales quotas, etc.) can be described. In the general form of fuzzy queries (2), the fuzzy equality sign "= ~" is used to describe linguistic variables in queries. For example[22],

assortment number = ~ more, expiration date = ~ less, price = ~ expensive

Linguistic variables (LV) are generally considered to be variables that take words and phrases as values, which in turn are described as fuzzy sets.

Definition 3. It is accepted to describe linguistic variables in the form of a cortege of the following parameters [2]:

$$\langle \beta, T, X, G, M \rangle \qquad (3)$$

, where:

- β - the name of the linguistic variable (for example: "**daily sales of the pharmacy**");
- T-terms of the linguistic variable (for example: **"less sales, better sales, more sales"**);
- X – is the domain of the linguistic variable (for example: «$0 \leq X \leq 30000$»);
- G (membership functions) - syntax procedures that facilitate the creation of new term sets using modifiers "MUCH", "NO", " LITTLE" (for example: "no less sales");
- M is the semantic procedure (mathematical rules that allow to find the type of membership functions corresponding to the term-sets formed on the basis of rules G (mathematical rules: fuzzy connectors (for example, connector "» "corresponds to A ∩ B, connector" or "corresponds to A ∪ B) and modifiers (for example, the modifier "a lot" corresponds to CON A = A2, and the modifier "a little" corresponds to DIL A = A0.5). we can.
  For example, the date of last use of a linguistic variable.

Expiration date = {less, good, more} (4)

Definition 4. A fuzzy query requires the ranking of the resulting elements - alternatives in accordance with the degree of their relevance to the query (usually a number from 0 to 1). In other words, it is necessary to sort the received result of the fuzzy query in such a way that it would be easier for the users of the queries to understand them.

Definithion 5. Triangular fuzzy numbers - most often used as predictive values of parameters and corresponds to a term-set for example T = {x is approximately equal to x*}. It is clear that $x^* \pm \Delta x \approx x^*$, and as Δx decreases to zero, the degree of confidence in the estimate grows to unity. This, from the point of view of the membership function, gives the latter a triangular appearance (Figure 1), and the degree of approximation is characterized by an expert. A more general form of a triangular fuzzy number can be described by the following triplet (x_1, x_2, x_3), where the membership can be determined as the following equation, which can be shown in Fig. 1.

$$\mu(x) = \begin{cases} 0 & 0 < x \le x_1 \\ \frac{x-x_1}{x_2-x_1}, & x_1 \le x \le x_2 \\ \frac{x_3-x}{x_3-x_2}, & x_2 \le x \le x_3 \\ 0, & x_3 \le x \end{cases} \quad (5)$$

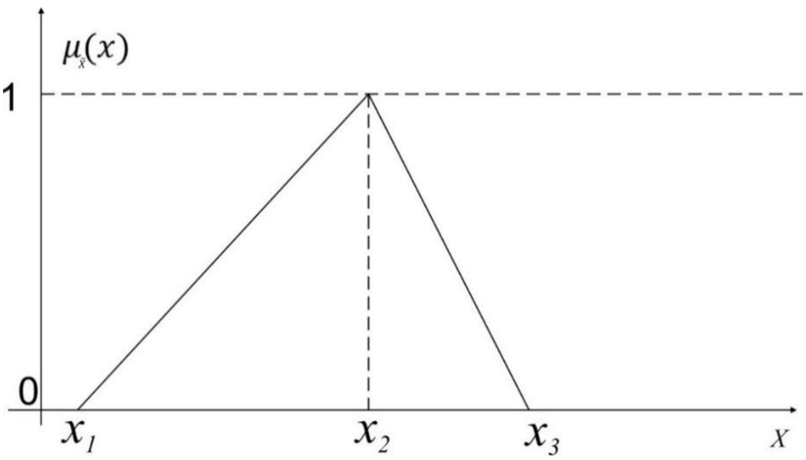

Fig. 1: A triangular fuzzy number.

Definition 6. Confidence interval and risks. Let us assume that N (x) is a fuzzy range of values x with a membership function $N_\alpha(x)$, and $N_\alpha(x)$ is a fuzzy set of level α. Then they say that α is the level of confidence, and N_α is the interval of the α -level of confidence, the confidence interval of the level of α, or the α-cut of the fuzzy value x. Here N_α is the interval of possible values of x, the degree of membership of which is equal to or greater than α.

Confidence intervals of the α-level can be associated with the indicator of the risk-content of a fuzzy value [3].

Definition 7. Fuzzy Z –number. A fuzzy Z-number is a tuple consisting of two fuzzy numbers Z = ($\tilde{X}$, $\tilde{R}_x$) [3,8,17,18,19], where $\tilde{X}$ is a fuzzy number presented by an expert to assess the factor under investigation; $\tilde{R}_x$ is a fuzzy number describing the degree of confidence in the expert's assessment (Figure 2).

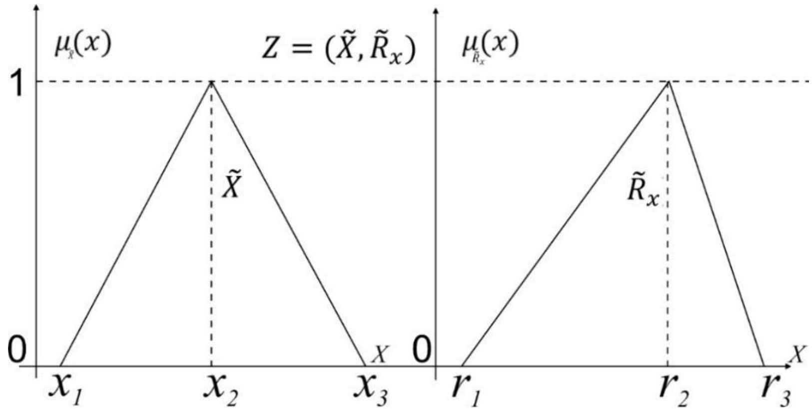

Fig. 2. Graphical representation of a fuzzy z-number

3 Problem statement

Currently, there is no single method for evaluating and ranking the results of fuzzy queries.

A fuzzy query has a large number of alternatives that must be evaluated according to fuzzy criteria, which must be represented as fuzzy Z-numbers.

Let's say there is a fuzzy query with the following content: **Make a list of drugs that are close to expiration date and are selling poorly.** Here we find fuzzy classes [22]: QS1: Goods LV [expiration date, less], QS2: Goods LV [sales, bad]. At the same time, when you ask the manager what to take as a basis, he says that drugs with a shorter shelf life are of most interest to us, but for greater confidence, it is also necessary to consider the poor sales of the drug.

As can be seen from the request, it is also necessary to evaluate the reliability of fuzzy conditions - fuzzy associations [22] This request in the form of Z-numbers can be represented as follows:

-Z_1 = (the expiration date of the medicine is less than a month, the medicine with a shorter shelf life, I am sure that this medicine needs to be returned to the distributor).

-Z_2= (the medicine sold less than 10 pieces in a month, the medicine is not selling well, I am very sure that this medicine needs to be returned to the distributor).

What to say about the fact that the manager is sure that drugs with shorter shelf life must be returned first of all, but among them there may be drugs that are selling very well. Therefore, for greater confidence, it is necessary to choose medicines with poor sales. Usually, the number of drugs in the databases is over 40-50 thousand. Therefore, it is almost impossible to evaluate every medicine by expert means. In this connection, it is necessary to develop an approach with the help of which it was possible, without the intervention of experts, to formulate a fuzzy Z-number in a dynamic mode.

The peculiarity of relational queries lies in the nature of the connection of relational tables, which depends on the formulation of the query itself. For our case, the table is taken as a basis, where information about the expiration dates of drugs is stored, which is linked through the LEFT OUTER JOIN with a relational table where the movements of goods are stored, etc.

The next section proposes an approach that transforms fuzzy query conditions to Z-numbers, which allows multi-criteria decision-making techniques to be applied to evaluate and rank the results of fuzzy queries.

4 The Proposed Method of Converting Regular Fuzzy Number to Z-number

Based on the statement "a fuzzy query is a kind of flexible query, in which the user describes the requirements with fuzzy query conditions" [1], a method is proposed to formalize fuzzy query conditions in the form of a Z-number.

As can be seen from definition 7, a fuzzy Z-number consists of $\tilde{X}$– fuzzy number, which, when solving problems of multi-criteria decision-making, are presented by experts to assess the factor under study; $\tilde{R}_x$ – is a fuzzy number describing the degree of confidence in the expert's assessment.

Select $\boldsymbol{\pi}_{(*)}$ **from tables where** $\oplus Z_{ij} = \left(\widetilde{X_{ij}},\ \tilde{R}^{ij}_{\tilde{x}}\right); i = \overline{1, N},\ j = \overline{1, M}$ (6)

Here, the tables list the tables used in the query $\boldsymbol{\pi}$ **(*)** are the projections of the columns of those tables, $\boldsymbol{Z_{ij}}$is the transformation of fuzzy conditions into Z-numbers, N is the number of alternatives in the query, M is the number of conditions, and ⊕ is the logical operation between fuzzy conditions..

a. Formation of the first part of the Z-number $\tilde{X}$- fuzzy number.

Managers usually use generalized interval values when determining the parameters of goods. For example, goods with a low price - goods with a price of 1 to 10 manat, goods with a small sales volume of 0 to 10 manat per day, goods with a short expiration date of 0 to 30 days, and so on. Thus, if we take into account the real prices of the goods participating in the survey, then each parameter can be described in the form of triangular fuzzy numbers - (x_1, x_2, x_3) (Figure 2). Here x2 is understood as the expected value. In other words, managers simulate statements of the type: "Parameter X is equal to x2 and is in the interval $[x_1, x_3]$", writing as $\tilde{X} \cong [\ x_1, x_2, x_3\ ]$. In this case, the whole range is called the universe, and the number x_*- supremum.

For example , x= x* və μ(x*)/x*=∝.

In this case, the main task is to form a fuzzy number $\tilde{X}$ to solve the problem posed in the article. The following algorithm is proposed for this::

1. Determining to which term-set x = x* belongs. For this case, the term-set with the maximum value among the values of the membership functions for all term-sets is found according to the following formula (Figure 3):

$$\alpha^t_{x*} = \max\left(\sum_{i=1}^{n} \frac{\mu^{t_i}(x*)}{x*}\right) \quad (7)$$

where ti are term-sets, $\mu^{t_i}(\ x^*)$ are the values of the membership function for each term-set and x^*.

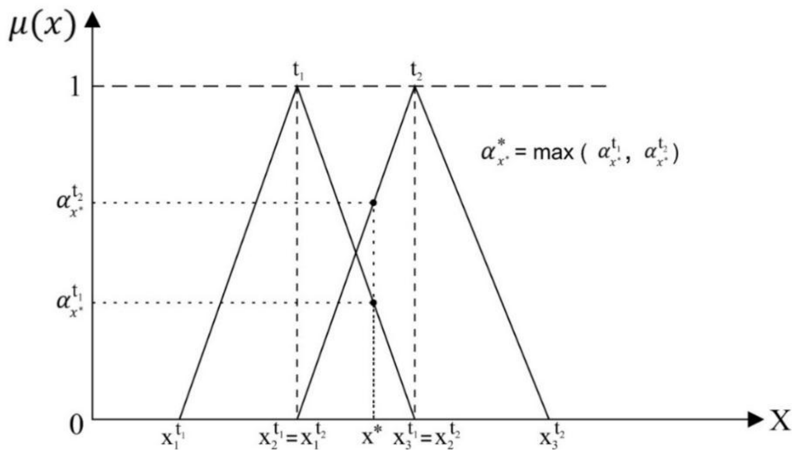

Fig. 3. Determining to which term – set x = x^* belongs

As shown in Figure 4 for the example, if the value of x=x_* then $\alpha^t_{x_*}$ then it is safe to say that x=x^* t with a small number of sales term-plurality and QS: Goods belong to the fuzzy class [sales, less].

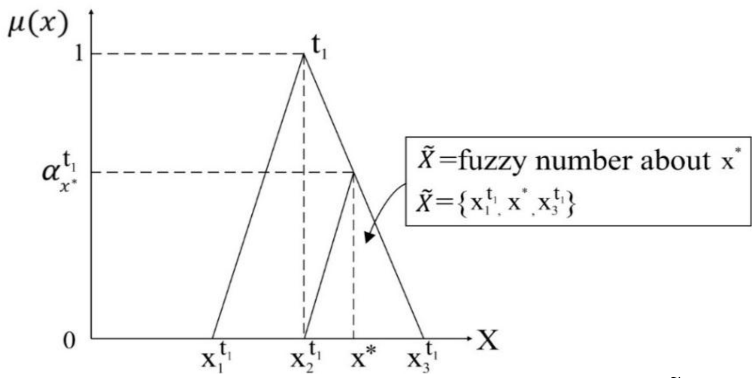

Fig. 4. Graphical representation of the fuzzy number $\tilde{X}$

2. Calculation of the initial and final boundary interval values of the fuzzy number $\tilde{X}$. When x = x *, one of the main problems is to find the last boundary interval points of the fuzzy number $\tilde{X}$, if it is certain that $\alpha^t_{x_*}$ belongs to the t-term-set of x (Figure 4).

First of all, we know that when x = x*, the value obtained by the membership function is α^t_{x*} . And if $\alpha^t_{x*} < 1$, it is a violation of the principle of normalization of the fuzzy number $\tilde{X}$. But on the other hand, the main feature of fuzzy queries is that it is known in advance that x = x*, ie for the value of x* of the triangular fuzzy number $\tilde{X}$

$\alpha^{t}_{x*} = \mu(x^{*})/x^{*} = 1$ and refers to the grass-term-set. To do this, it is suggested to use the following formulas [21], which are most commonly used in practice:

$$x'_1 = x_1 + \alpha^{t}_{x_*}(x_* - x_1)\ ;\ x'_3 = x_3 - \alpha^{t}_{x_*}(x_3 - x_*) \quad (8)$$

The main issue here is the risk of accepting the triangular fuzzy $\tilde{X}$ in these intervals and at the expected value. That is, what percentage can be sure that the newly found - formed $\tilde{X}$ fuzzy number x = x* is true: "The parameter x is equal to x* and is in the interval [x'_1, x'_3]- confidence and risk can be judged based on the value of the membership function (definition 6). In other words, if . α^{t}_{x*} is an confidence indicator, then $1 - \alpha^{t}_{x*}$ can be considered as a trisk indicator (uncertainty). To do this, the following rule is proposed in []: First, a universal formula for calculating x is proposed:

$$x_u = \frac{2x^{*}x'_3 - x'_1x'_3 - (x^{*})^2}{x'_3 - x'_1} \quad (9)$$

and the value of the membership function for x_u is calculated::

$$\propto (x_u) = \frac{x_u - x^{*}}{x'_3 - x'_1} \quad (10)$$

The universality of formula 9 is that in the values found on its basis, the values of the belonging function of the fuzzy number x overlap at the boundaries of the numbers x <x * and x> x *.

Then, based on the formula of an integral measure of the probability of the accepted fuzzy number:

$$r(\propto (x_u)) = \frac{\propto (x_u)^2}{2} * 100 \quad (11)$$

the risk rate (percentage) is calculated.

b. $\tilde{R}_x$ - of the second part of the Z-number - the formation of a fuzzy number. Determination of reliability describing the degree of confidence in the expert's assessment.

In our case, the expert is an algorithm for forming $\tilde{R}_x$ - a fuzzy number, which is described below:

1. Confidence intervals of the α-level can be associated with the index of risk-content of a fuzzy quantity [3] In other words, α-levels of a fuzzy set can also be associated with the reliability of the statement that, for example, "Parameter X is equal to x_* with reliability $\tilde{R}_x$ and is in the interval $[x_1, x_3]$", where $\tilde{R}_x$ is a fuzzy number.

   To define a fuzzy number - the reliability of $\tilde{R}_x$, you can use definition 2 in [8], where five linguistic concepts are given to describe the degree of safety of the R-universe of discourse - R = {Very Low, Low, Medium, High, Very High}, assuming that that only two adjacent linguistic variables have the same value. The mathematical formulas for calculating the value of the membership function for each linguistic variable and their graphical representations are given (Figure 5). Figure 5 was modified by us to determine the intervals according to the α = 0.5 - section. This definition can also be used for our case - to determine the degree of confidence that the parameter X is equal to x^{*} with the reliability $\tilde{R}_x$., according to the following algorithm.

2. According to the value obtained in step a, step 2, the value $\propto (x_u)$, using the formulas proposed by us below, the fuzzy number of the number $\tilde{R}_x$ is determined:

$$\tilde{R}_{x\cdot} = \begin{cases} VL \quad [0,0,0.125] & where \quad 0.000 \leq \propto (x_u) \leq 0.125 \\ L \quad [0.125,0.25,0.375] & where \quad 0.125 < \propto (x_u) \leq 0.375 \\ M \quad [0.375,0.500,0.625] & where \quad 0.375 < \propto (x_u) \leq 0.625 \\ H \quad [0.625,0.25,0.875] & where \quad 0.625 < \propto (x_u) \leq 0.875 \\ VH \quad [0.875,1,1] & where \quad 0.875 < \propto (x_u) \leq 1.000 \end{cases} \tag{12}$$

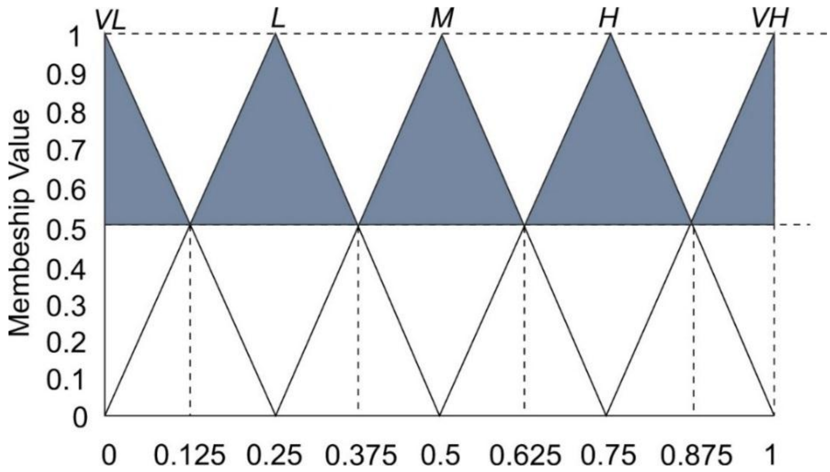

Fig. 5: Fuzzy condition reliability membership function

c. Formation of Z-number.

As a result of the calculations performed on items a and b, each condition and one alternative given in the fuzzy query is converted to the following Z_{ij}numbers::

$$Z_{ij} = (\widetilde{X_{ij}},\ \tilde{R}_{\tilde{x}}^{ij}); i = \overleftarrow{1, N},\ j = \overleftarrow{1, M} \tag{13}$$

, where N is the number of alternatives in the query and M is the number of conditions.

d. Evaluation and ranking of alternatives according to the given criteria and on the basis of Z-numbers.

Recently, there are many decision-making methods using Z-numbers under uncertain environments. Examples of these are [8,17,19]. Using these methods, alternatives can be very effectively evaluated and ranked according to criteria.

5 A Numerical Example.

Suppose that a fuzzy query is given: **Select * from items where price is middle**. Obviously, the number of goods here can be in the tens of thousands. Let's look at the formation of a Z-number on a particular commodity.

For example, let's take a term-set as the parameters of the membership function (10,50,100) as goods with average sales (Figure 6). Here, at the boundary values of 10 and 100, the values of the membership function are assumed to be 1 when 0, 50. The real sales volume for a particular concrete product is 25, while the value of its membership function is 0.6250 (Figure 6).

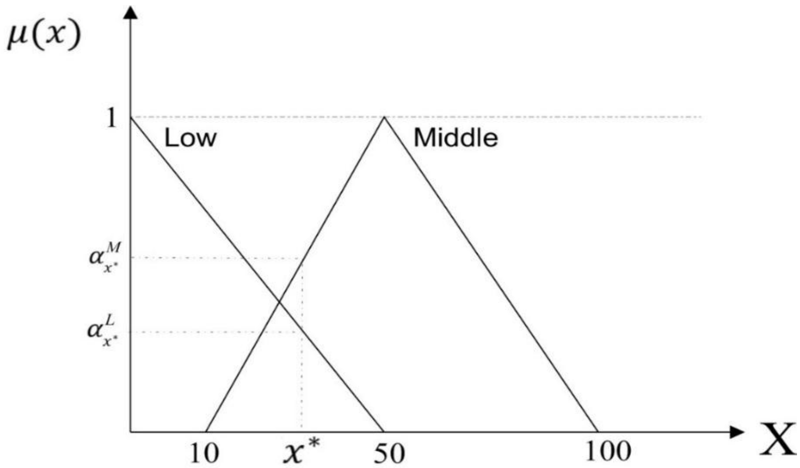

Fig. 6. Determining the term-set to which the number x* belongs

Under these conditions, the fuzzy $\tilde{X}$ will have the following boundary values:

According to the above formulas we get the following values: $x_1' = 35$, $x_3' = 68.75$, $x_u = 58.3334$, $\propto (x_u) = 0.5556$, $r(\propto (x_u)) = 0.1544 * 100 = 15.44\%$. So, if the real sales number for m-mali is 25, then with a risk of 15.44%, in other words with 84.56% confidence, $\tilde{X} = (35,50,68.75)$ can be accepted in the triangular fuzzy numerical reports. However, if we make a decision based on definition 4, then we should take 0.6250 * 100 = 62.50% and 37.50% as uncertainty, ie risk rate.

Now $\tilde{R}_x$. - form a fuzzy number. To do this, using the formula 12, we see that $0.375 < \propto (x_u) = 0.5556 \leq 0.625$. Hence $\tilde{R}_x = (0.375,0.500,0.625)$
And as a result we can form the Z-number as follows:

$$Z = (\tilde{X} = (35,50,68.75), \ \tilde{R}_x = (0.375,0.500,0.625))$$

By the same calculation methods, all alternatives are converted to Z-numbers and a decision-making matrix is constructed. In the next stages, Z-numbers are formed on the criteria and goods-alternatives required in the survey, and evaluation and ranking operations are carried out on the basis of Z-numbers using application methods.

6 Conclusion.

The proposed method allows, without the intervention of experts, to form fuzzy conditions of fuzzy queries in the form of a Z-number, which allows processing queries with a large number of records - alternatives.

However, it should be noted that there are some subject areas in which there are such parameters, where the first component of the Z-number is formulated as follows: $\tilde{X} \cong [\ x_1, x_2, x_3\ ]$ где $x_1 = x_2 = \ x_3$. For example, in cases where the prices of drugs are regulated by state tariff boards, prices are the same in all periods.

It should be noted that the main problem that remains unresolved in the processing of fuzzy requests is the solution of the problem of degradation of responses by relevance to the request. That is, it is necessary to conduct further research in the field of classification of results in order to determine the relevance of the results to the query. The solution to this problem will make it possible to answer such questions as to how much one alternative (answers of a fuzzy query) is better than the other. For example, if you need to know which of the answers are the best, the best, the worst, the worst, etc.

It is also necessary to consider the application of Z-number arithmetic when processing fuzzy queries.

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