Mathematical Modeling of Hoverboard

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Abstract. A self-balancing scooterate (also"hoverboard",self-balancingsystem on board) is a self-balancing transport system consisting of motorized wheels have pad on which the rider places her or his feet and stands on pad to drive. The driver or a person controls the velocity of Hoverboard by leaning forwards or backwards, and provide direction with a steering command. The study of balancing of a person on Hoverboard can be explained with the help of a complex computer algorithm that stabilizes the under-actuated system .

The methodology or The Concepts used for controlling the Hover board mechanism through mathematical modelling can be studied by explaining through kinematic model, the dynamic model (using Lagrange approach) that are presented to control the Two-Wheel personal Balancing Transporter (TWPBT) as follows.

1. INTRODUCTION

A self-balancing scooterate (also"hoverboard",self-balancingsystem on board) is a self-balancing transport system consisting of motorized wheels have pad on which the rider places her or his feet and stands on pad to drive. The driver or a person controls the velocity of Hoverboard by leaning forwards or backwards, and provide direction with a steering command. The study of balancing of a person on Hoverboard can be explained with the help of a complex computer algorithm that stabilizes the under-actuated system . With the consequence of back or forward movement ,The person can move system by leaning forwards or backwards and can use controller to stabilize it ,.
Since 2001, Segway PT are available in the market used as a two-wheeled self-balancing vehicles and recognized as a powerful personal transporter and commercial vehicle as a versions. Another successful example is -so called- Hover board : .The Hover board may be consider as an evolution of the first, it has drive mechanism and has the advantage of being minimum weight , portable and smaller in size.

Another section The problem of controlling the Two-Wheel Personal Balancing Transporter (TWPBT) starts with the mathematical modelling that controls the Energy which are govern by kinematic modelling , the dynamic modelling explained through Lagrange mathematical Equation .

A subsection Mathematical Modelling of Hover board

Some text. The methodology or The Concepts used for controlling the Hover board mechanism through mathematical modelling can be studied as follows. The problem of controlling the Two-Wheel Personal Balancing Transporter (TWPBT) starts with the mathematical modelling that controls the Energy which are govern by kinematic modelling , the dynamic modelling explained through Lagrange mathematical Equation are presented and explained in details as follows ;

2. **KINEMATIC MODELLING OF HOVERBOARD** .

The Nomenclature used for this Model are

Let

m_P Mass/weight of the Driver [kg]/N
m_w Mass/weight of the Right AND Left wheel (SAME) [kg/N]
$J\theta_P$ Mass moment of inertia of the Driver, w.r.t. pitch rotation [kgm^2]
$J\delta_P$ Mass moment of inertia of the Driver w.r.t. yaw rotation [kgm^2]
J_w Mass moment of inertia of the ROTOR(wheels) [kgm^2]
α_m,β_m Angular position of the (Right, Left) motor (w.r.t to the base) [rads]
α,β Angular position of the (Right, Left) ROTOR(wheels)(w.r.t to the ground) [rads]
θ_P Angular position of the Driver (w.r.t .the ground, where 0 is the upper position in the vertical direction) [rads]
v_L,v_R Linear Velocity of the (Right, Left) centre of the ROTOR(wheels) [m/s]
x_b,v_b X-axis position and Linear Velocity of the centre of the base (origin) [m],[m/s]
x_P,y_P,z_P – X-Y-Z-coordinates of the Driver's centre of Mass [m]
L Length between centre of Mass of the Driver and Base [m]
D Distance between ROTOR(wheels) [m]
R Radius of the ROTOR(wheels) [m]
C_L,C_R Torques acted on the ROTOR(wheels), by Motor (than to gearbox) [Nm]
ρ Velocity ratio of the motor and the wheel
τ_L,τ_R Torque of the (Right, Left) motor [Nm]
ψ Frictional viscosity coefficient

ABOVE Nomenclature gives Meaning with units for Hoverboard

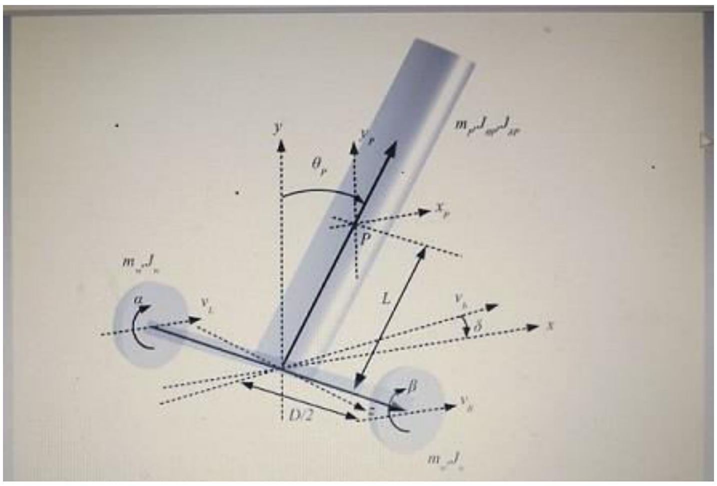

Figure 1: Graphical representation of Hover board showing above parameters

3 ASSUME SUITABLE DATA

-Let's assume suitable data:

1 The frictional parameter is assumed to be linear and directly proportional to the motor's velocity, other than reality.

2. Assuming Driver as a rigid body (cylinder) of "2L" height

3. Efficiency of Gearbox = 1

4. No Elasticity to Velocity ratio

5 Neglecting Air friction

6. System's vertical origin is treated as vertical coordinate of the base

The given below equation relates motor's position (declared to the Driver's angle) with the ROTOR's(wheels) angle (declared to the ground) are :

$$\alpha = \theta_P + \rho\alpha_m$$
$$\beta = \theta_P + \rho\beta_m$$
$$\dot{\alpha} = \dot{\theta}_P + \rho\dot{\alpha}_m$$
$$\dot{\beta} = \dot{\theta}_P + \rho\dot{\beta}_m \qquad (1)$$

The relation between the ROTOR(wheels) and base are:

$$v_L = r\dot{\alpha}$$
$$v_R = r\dot{\beta}$$
$$v_b = v_L + v_R\ 2r = r\ \dot{\alpha} + \dot{\beta}\ 2r$$
$$\delta = v_L - v_R\ D = r\ \dot{\alpha} - \dot{\beta}\ D \qquad (2)$$

Referring to Driver's centre of mass:

$$x_P = xb + L\sin\theta P\cos\delta$$
$$y_P = L\cos\theta P$$
$$z_P = zb + L\sin\theta P\sin\delta$$
$$\dot{x}_P = \dot{x}b + L\dot{\theta}P\cos\theta P\cos\delta - L\dot{\delta}\sin\theta P\sin\delta$$
$$\dot{y}_P = L\dot{\theta}P\sin\theta P$$
$$\dot{z}_P = \dot{z}b + L\dot{\theta}P\cos\theta P\sin\delta + L\dot{\delta}\sin\theta P\cos\delta$$
$$v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 =$$
$$= v2\ b + L2\ \dot{\theta}2\ P + 2L\dot{\theta}P[\dot{x}b\cos\theta P\cos\delta + \dot{z}b\cos\theta P\sin\delta] + +2L\dot{\delta}[-\dot{x}b\sin\theta P\sin\delta + \dot{z}b\sin\theta P\cos\delta \qquad (3)$$

where:

$\dot{x}_b = v_b\cos\delta$ $\dot{z}_b = v_b\sin\delta$ (4)

thus:

$v2\ P = v2\ b + L2\ \dot{}\ \theta 2\ P + 2L\dot{}\ \theta_P[v_b\cos\theta_P\cos 2\delta + v_b\cos\theta_P\sin 2\delta] =$

$= v2\ b + L2\ \dot{}\ \theta 2\ P + 2L\dot{}\ \theta_P v_b\cos\theta_P$ (5)

4 DYNAMIC MODELLING OF HOVER BOARD

- **The Resultant torque**, of motor after the gearbox, due to Frictional viscosity coefficient can be modelled as:

$C_L = 1/\ \rho\ (\tau_L - \psi\dot{}\ \alpha_m) = \quad 1/\ \rho\ \tau_L - \psi\ \rho 2\ (\dot{}\ \alpha - \dot{}\ \theta_P)$

$C_R = 1/\ \rho\ (\tau_R - \psi\dot{}\ \beta_m) = \quad 1/\ \rho\ \tau_R - \psi\ \rho 2\ (\dot{}\ \beta - \dot{}\ \theta_P)$ (1)

4.1 kinetic energy of the Rotors (wheel):

Neglecting the kinetic energy of rotor(wheel) about its vertical axis ,The equations given below referred both translational (linear)and rotational parameters of the rotor(wheel)motion.

$KE_L = 1\ /2mwv^2\ _L + 1/\ 2Jw\ \dot{}\ \alpha 2 = 1\ /2(mwr^2 + Jw)\dot{}\ \alpha^2$

$KE_R = 1\ /2mwv^2\ _R + 1\ /2Jw\ \dot{}\ \beta 2 = 1\ /2(mwr^2 + Jw)\dot{}\ \beta^2$

$KE_w = T_L + T_R = 1/\ 2(mwr^2 + Jw)(\dot{}\ \alpha + \dot{}\ \beta)^2$ (2)

4.2 KINETIC ENERGY OF THE DRIVER:

the **kinetic energy** of the Driver is compounded with three parameters as :

Thus **translational or the Linear kinetic energy** is written as :

$\mathbf{KE_P} = 1/\ 2m_Pv^2\ _P = 1/\ 2m_P(v^2\ _b + L2\ \dot{}\ \theta 2\ P + 2L\dot{}\ \theta_P v_b\cos\theta_P)$

$= 1\ /2m_P[r\ \dot{}\ \{\alpha + \dot{}\ \beta\ /2\ \}2 + L2\ \dot{}\ \theta 2\ P + 2L\dot{}\ \theta Pr\ \dot{}\ \alpha + \dot{}\ \beta\ 2\ \cos\theta_P]$ (3)

The **rotational or circular kinetic energy** w.r.t. rotation of the Driver around the rotor's (wheel's) centre (θ_P) is written as:

$KE\theta_P = 1/\ 2J\theta_P\ \dot{}\ \theta 2\ _P$ (4)

The **rotational or circular kinetic energy** w.r.t the rotation around the vertical axis (y) is written as

$KEy_P = 1/\ 2(\mathbf{J}\delta_P + m_PL2\sin\theta 2\ _P)\dot{}\ \delta 2$

$= 1\ /2(J\delta_P + m_PL2\sin\theta 2\ _P)\{r\ \dot{}\ \alpha - \dot{}\ \beta\ /D\ \}2$ (5)

Therefore **Total kinetic energy of the Driver** is given by:

$$KE_P = KEt_P + KE\theta_P + KEy_P$$
$$= 1/2m_P[\{r\dot{\alpha} + \dot{\beta}^2\}^2 + L^2\dot{\theta}^2_P + 2L\dot{\theta}_P r\dot{\alpha} + \dot{\beta}^2\cos\theta_P] +$$
$$1/2J\theta P\,\dot{\theta}^2_P +$$
$$1/2(J\delta_P + m_P L^2\sin^2\theta_P)\{r\dot{\alpha} - \dot{\beta}/D\}^2 \quad (6)$$

4.3 KINETIC ENERGY OF THE MOTORS:

The kinetic energy of the motors is:

$$KEm = KEmL + KEmR = 1/2Jm(\dot{\alpha}^2 m + \dot{\beta}^2 m) =$$
$$= 1/2\, Jm/\rho^2(\dot{\alpha}^2 + \dot{\beta}^2 + 2\dot{\theta}^2 P - 2\dot{\alpha}\dot{\theta}_P - 2\dot{\beta}\dot{\theta}_P) \quad (7)$$

Total kinetic energy:

Therefore Summation of all kinetic energy of the system is given as :

$$KE = KE_P + KE_w + KEm =$$
$$1/2m_P[\{r\dot{\alpha} + \dot{\beta}^2\}^2 + L^2\dot{\theta}^2_P + 2L\dot{\theta}Pr\dot{\alpha} + \dot{\beta}^2\cos\theta_P] +$$
$$\tfrac{1}{2}(J\theta_P\dot{\theta}^2 P +$$
$$1/2(J\delta_P + m_P L^2\sin^2\theta_P)\ \{r\dot{\alpha} - \dot{\beta}/D\}^2 +$$
$$1/2(m_w r^2 + J_w)(\dot{\alpha} + \dot{\beta})^2 +$$
$$1/2\, Jm\,\rho^2(\dot{\alpha}^2 + \dot{\beta}^2 + 2\dot{\theta}^2 P - 2\dot{\alpha}\dot{\theta}_P - 2\dot{\beta}\dot{\theta}_P) \quad (8)$$

4.4 Potential energy (of the Driver):

The potential energy w.r.t the vertical position of the Driver's center of mass:

$$PE = m_P g L\cos\theta_P \quad (9)$$

5 **LAGRANGIAN EQUATION**

USING Lagrangian rule, the Equation for the system can be written as

$$T = KE - PE$$

Where T = Total Energy or Torque

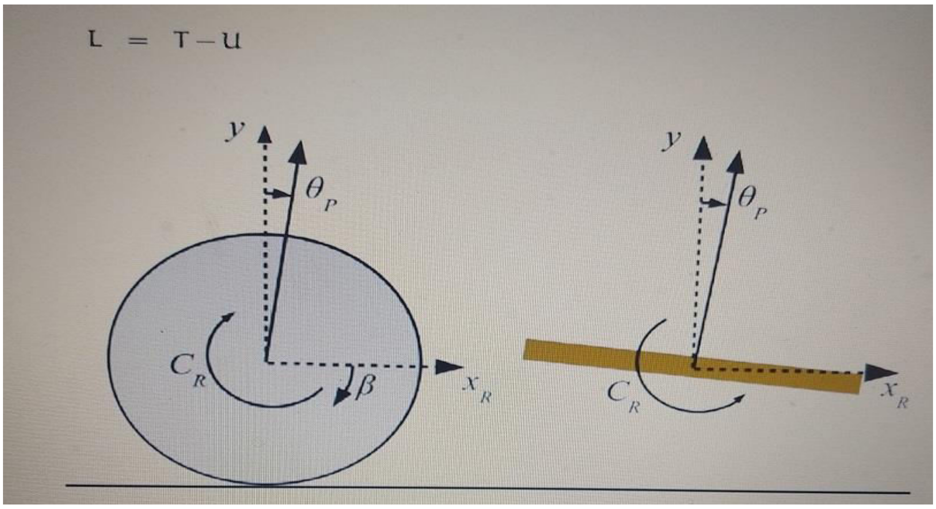

Fig : 2 Torque acting on a wheel

The above figure gives the total idea how torques is applied on left wheel with complete references as shown., while in the left of the TWPBT base: and In the Right, the rotor(wheel),,,it is to be noted that the torque generated by the motor applies equal and opposite in the two bodies.
The symmetric holds for the right rotor(wheel).

W.R.T ABOVE Situation, Therefore the Lagrange equations can be mathematically modelled as :

$$d/dt(\partial L\, \partial \dot{}\, \alpha) - \partial L/\partial\alpha = C_R$$
$$d/dt(\partial L\, \partial \dot{}\, \beta) - \partial L/\partial\beta = C_L$$
$$d/dt(\partial L\, \partial \dot{}\, \theta P) - \partial L/\partial\theta P = -(C_R + C_L) \quad (1)$$

CONCLUSIONS :

1) The right-side of the Lagrange equations state the active contribution to the dynamics.
2) The first and second equations' terms are torques of the motors,
3) the third equation, the active contribute is the sum of the torques of the two motors, as easily noticeable

Thus permits to write the complete Mathematical Modelled Equations of motion as

$[m_P r2/4 + r2(J\delta P + m_P L2 \sin 2\theta_P)/D2 + mwr2 + Jw + Jm/\rho 2]\ddot{}\ \alpha +$
$[m_P r2/4 - r2(J\delta P + m_P L2 \sin 2\ \theta_P)/D2 + mwr2 + Jw]\ddot{}\ \beta +$
$[m_P Lr\cos\theta_P/2 - Jm/\rho 2]\ddot{}\ \theta_P + \psi\ \rho 2\ \dot{}\ \alpha - \psi\ \rho 2\ \dot{}\ \theta_P +$
$-m_P L\ \dot{}\ \theta 2\ Prsin\ \theta_P/2 + 2m_P L2r2 \sin\theta_P \cos\theta_P/D2\ \dot{}\ \theta P(\dot{}\ \alpha - \dot{}\ \beta) = \tau L/\rho$
$[m_P r2/4 - r2(J\delta P + mPL2 \sin 2\ \theta_P)/D2 + mwr2 + Jw]\ddot{}\ \alpha +$
$[m_P r2/4 + r2(J\delta P + mPL2 \sin 2\ \theta_P)/D2 + mwr2 + Jw + Jm/\rho 2]\ddot{}\ \beta +$
$[m_P Lr\cos\theta_P/2 - Jm\ \rho 2]\ddot{}\ \theta_P + \psi/\rho 2\ \dot{}\ \beta - \psi/\rho 2\ \dot{}\ \theta P +$
$-m_P L\ \dot{}\ \theta 2\ Prsin\ \theta_P\ 2 + 2mPL2r2\sin\theta P\cos\theta P\ D2\ \dot{}\ \theta P\dot{}\ \beta - \dot{}\ \alpha = \tau R/\rho$
$[m_P Lr\cos\theta_P/2 - Jm\ \rho 2]\ddot{}\ \alpha + [mPLr\cos\theta_P\ 2 - Jm/\rho 2]\ddot{}\ \beta +$
$+[m_P L2 + 2Jm\ \rho 2 + J\ \theta_P]\ddot{}\ \theta_P - m_P gL\sin\theta P +$
$+2\psi/2\ \dot{}\ \theta_P - \psi/\rho 2\ \dot{}\ \alpha - \psi/\rho 2\ \dot{}\ \beta - mPL2r2\sin\theta_P\cos\theta_P/D2\ (\dot{}\ \alpha - \dot{}\ \beta)2 = -\tau L/\rho - \tau R/\rho$

Above are the **equations of motion** for the Hover board system.

In order to make a linear controller, it is possible to linearize system into a non-linear system

6. FUTURE SCOPE

Self-balancing scooterate system like Hoverboard is the simplest mathematical model for the study.The Validation of above mathematical model can be made possible using simulation software like simulink block model.And therefore the parameters related to System's,Subsystem (example:stability,poles etc) can easily be identify for making the model usable for design of suitable and proper controller. The system can be made more realistic if one can go for study related to model extentions ,which will more exhaustive and interesting.

The evoluted model can also be analyzed from the above mathematical model parameters, where Driver will have few degree of freedom w.r.t. the angle of the base using above references.
Using model's parameters regarding the Driver and its nature of driving –A Block model can be prepared using physiological parameter. The mathematical model presented as a system model is used for the "self-balancing Robotised System" problems, that consist of a rigid body erected on two wheels.

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References

[1] Sun, Z. Yu, and H. Yang. A design for two-wheeled self-balancing robot based on kalman filter and lqr. In 2014 International Conference on Mechatronics and Control (ICMC), pages 612–616, July 2014. doi:10.1109/ICMC.2014. 7231628

[2] Mansoor,I.U.H.shaikh,andS.Habib. Genetic algorithm based lqr control of hovercraft. In 2016 International Conference on Intelligent Systems Engineering (ICISE), pages 335–339, Jan 2016. doi: 10.1109/INTELSE.2016.7475145

[3] J. R. Cao, C. P. Huang, and J. C. Hung. Stabilizing controller design using fuzzy t-s model on two wheeled self-balancing vehicle. In 2016 International Conference on Advanced Materials for Science and Engineering (ICAMSE), pages 520–523, Nov 2016. doi: 10.1109/ICAMSE.2016.7840187.

[4] Y. Gong, X. Wu, and H. Ma. Research on control strategy of two-wheeled self-balancing robot. In 2015 International Conference on Computer Science and Mechanical Automation (CSMA), pages 281–284, Oct 2015. doi: 10.1109/CSMA. 2015.63.

[5] B. P. Kumar and Krishnan C. M. C. Comparative study of different control algorithms on brushless dc motors. In 2016 Biennial International Conference on Power and Energy Systems: Towards Sustainable Energy (PESTSE), pages 1–5, Jan 2016. doi: 10.1109/PESTSE.2016.7516444.

[6] Winter, D., Patla, A., Prince, F., Ishac, M., and Gielo-Perczak, K., Stiness control of balance in quiet standing,Journal of Neurophysiology, vol. 80, no. 3, pp. 12111221, 1998.

[7] Kuhn T 1998 *Density matrix theory of coherent ultrafast dynamics Theory of Transport Properties of Semiconductor Nanostructures* (Electronic Materials vol 4) ed E Schöll (London: Chapman and Hall) chapter 6 pp 173–214

[8]Pippard, A. B., The inverted pendulum,European Journal of Physics, vol. 8, no. 3, p. 203, 1987

[9]Model-based development of a self-balancing, two-wheel transporter by Relatore: Roberto Oboe Laureando: Dino Spiller

[10]Wikipedia the hoverboard. Self balancing scooter (hoverboard). URL https: //en.wikipedia.org/wiki/Self-balancing_scooter s