Stabilization and tracking enhancement of the ball on the plate system based on Pseudo-PD controller and machine learning algorithms.

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Abstract. This paper presents a novel method to improve the stabilization and trajectory tracking of the ball on the plate system (BOPS) based on machine learning algorithm with the Pseudo proportional-derivative (PPD) controller. The proposed controller depends on a machine learning (ML) algorithm that detect the angle of the servo motor required to correct the position of the ball on the plate. This paper presents three different ML algorithms for the servo motor angle prediction and achieved higher accuracy which are 99.85%, 100%, and 99.998% for support vector regression, decision tree regression, and random forest regression, respectively. The simulation results show that the proposed method has significantly improved the settling time and overshoot of the system. The mathematical formulation can be obtained using the Lagrangian formulation and the servo motor parameter obtained by a practical identification experiment.

1. Introduction

Balancing systems is one of the most challenging systems in the control fields [1]. There are some classical examples of balancing systems such as inverted pendulum [2,3] and ball and beam [4,5]. Ball on plate is an enhanced version of the ball and beam system which its plate consists of two perpendicular directions (x and y). One of the most important problems in control is balancing, tracking periodic references, and rejecting periodic disturbances. The position of the ball on plate is tracked by using a resistive touch screen [6] or image processing [7] as sensors. The image processing sensor is better than the resistive touch screen in terms of the position's sensitivity, which means that using the image processing as a sensor in the ball on the plate reduce the noise of the system. The actuator of the system can be made by stepper motors [8] or Servo motors [9]. The servo motor is commonly used to avoid the necessity of stepper driver circuit, encoder coupling, and the torque-speed curve of the stepper motor shows that the torque decreases with an increase of speed. To add things up, steppers consume current continuously even when idle unless it is controlled not to do so. The point stabilization and trajectory tracking problems are clearly shown in the ball on plate systems. The complete system is shown in Fig. 1. Several papers discuss these problems through modelling methods that neglect the non-linear

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behaviour of the system especially the relations between motor and plate angles in case of the range of plate angle is small [10]. But it was found that when the non-linear part of the relationship between the angle of the ball and the angle of the plate is neglected, the settling time of the BOPS is large and the overshoot.

Faiber et al. [11] presented an approach for BOPS based on fuzzy and PID controllers. This controller presented a settling time of 4 second an overshoot 15%. Agung Adiprasetya et al. [12] presented a PD algorithm with a pre-filter. This algorithm presented a large settling time and overshoot. Ibrahim Mustafa Mehedi et al. [13] presented fractional controller design for the ball and beam system. This algorithm presented a settling time of 4.5 second. Nearly all the published paper related to BOPS are linearized the relation between the servo motor angle and the plate angle.

This paper introduces the Pseudo-PD controller with three ML algorithms that deal with the nonlinear relation between the servo motor and the plate angle. The ML algorithms take the plate angle and detect the best servo motor angle to correct the ball position with a small settling time and overshoot.

The conventional PID controllers are still widely used in most industrial process. This is mainly because PID controller have simple control structure and its effectiveness for linear control systems [11]. The results shows that the ball point stabilization is performed without steady state error in case Pseudo-PD and Fuzzy but fuzzy decrease the overshoot and make the system faster. When using the lookup table with fuzzy the results is improved.

The rest of the paper is organized as follows. Section 2 mathematical model. Section 3 includes results and discussion. Finally, the conclusions are given in Sect. 4.

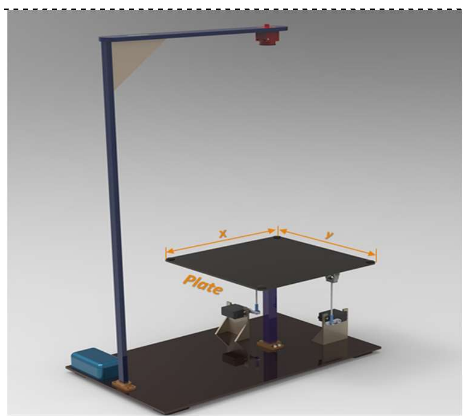

Figure 1. The design of BOPS.

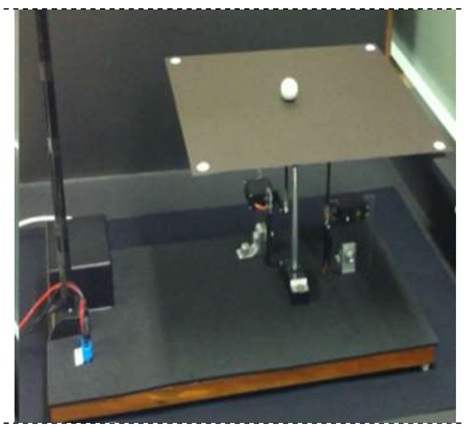

Figure 2. Ball on the plate system used for the experiment.

2. **Mathematical Model of the BOPS.**

The ball on the plate system consists of two servo motors (FEETECH FS511M) to rotate the acrylic plate in the X and Y direction by a specific angle. This system is used as an experimental prototype to investigate the proposed controllers to the ball and plate system as shown in Fig. 2. On top of the plate, there is a plastic free-rolling ball. The system is equipped with a camera that responsible for real-time images for sensing the ball position in the X and Y directions. All image processing and control algorithms are executed in the python language. To make the servo motor rotate, the ML algorithm computes the desired angles and sends them via serial port to the interface circuit that generates the required PWM signals for the actuators. The complete system is represented as shown in Fig.3.

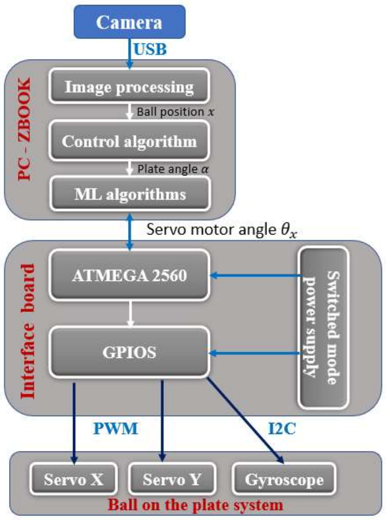

Figure 3. The complete system block diagram.

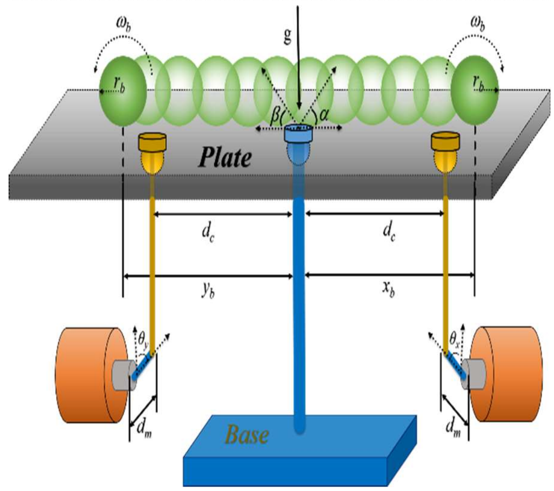

Figure 4. Schematic of Ball on the plate system X and Y-axis

A state-space model of the system is obtained to design the required controller. The modeling procedure represents the two-dimensional ball and plate system as two uni-dimensional decoupled ball and beam systems: one in the X direction and the other in the Y direction. Fig.4 represents the schematic of the ball on the plate system with its corresponding servo motors in terms of X and Y directions. The initial position of the plate is in a horizontal situation without inclination; which means that both angels θ_x and θ_y of the servo motors are equal to zero. On other hand, both the angles between the plate and the horizontal axes α and β are equal to zero. In terms of Fig.4, d_c is the distance between the plate central joint and the vertical arm joint, d_m is the servo motor arm length, x_b is the position of the ball in X direction, y_b is the position of the ball in Y direction, and r_b is the radius of the ball. The ball equation of motion can be obtained by using Lagrangian method.

The Euler-Lagrange equation of ball on the plate system derives as follows [1]:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \quad (1)$$

Where L is the Lagrangian and Q_i in (1) is the external forces acting on the system on q_i which is x in this case. This equation can be reformulated in terms of **Error! Reference source not found.** as follows:

$$L(q_i, \dot{q}_i) = T(q_i, \dot{q}_i) - V(q_i) \quad (2)$$

Where T is the total kinetic energy of the system since V is its total potential energy. Both are functions in terms of the generalized coordinate q_i and its derivatives $\dot{q}_i$. The total kinetic energy T of the ball can be calculated by the sum of translational kinetic energy T_t and rotational kinetic energy T_r .
The translational kinetic energy T_t is expressed as follows:

$$T_t = \frac{1}{2} m_b \dot{x_b}^2 \quad (3)$$

Where is m_b is the ball mass, and $\dot{x_b}$ is the translational velocity along X-axis.
The rotational kinetic energy T_t is expressed as follows:

$$T_r = \frac{1}{2} I_b \omega_b^2 \quad (4)$$

Where is I_b is the ball moment of inertia, and ω_b is the angular velocity along X-axis. It is noticeable that I_b depend on the shape of the ball where $I_b = \frac{2}{5} m_b r_b^2$ in case of solid sphere and $I_b = \frac{2}{3} m_b r_b^2$ in case of a hollow sphere. where (r_b) is the ball mass.
The translational velocity is expressed in terms of angular velocity and ball mass as follows:

$$\dot{x_b} = r_b \omega_b \quad (5)$$

It is possible to rewrite T_r as a function of the linear velocity as follows:

$$T_r = \frac{1}{2} I_b \frac{\dot{x_b}^2}{r_b} \quad (6)$$

The total kinetic energy for the solid sphere and the hollow sphere balls are expressed respectively as follows:

$$T = \frac{7}{10} m_b \dot{x_b}^2 \quad (7)$$

$$T = \frac{5}{6} m_b \dot{x_b}^2 \quad (8)$$

There is no energy effect on the ball except the gravity as shown in Fig. 5, so the potential energy in X direction is expressed as follow:

$$V_b = m_b g x_b \sin\alpha \quad (9)$$

Where x_b is the ball position on the plate in the X direction, g is the constant of gravity acceleration, and the plate inclination angle. It is noticeable that $x_b \sin\alpha$ is the x component of the gravitational force.
By solving the Lagrange equation (1), the only composite force actuating on the q_i is the friction F_x which can be written $Q_i = F_x = -f_c \dot{x_b}$, and hence the equation of motion of the ball in case solid sphere is expressed as follows:

$$\frac{7}{5}\ddot{x_b} + \frac{f_c \dot{x_b}}{m_b} + g\sin\alpha = 0 \quad (10)$$

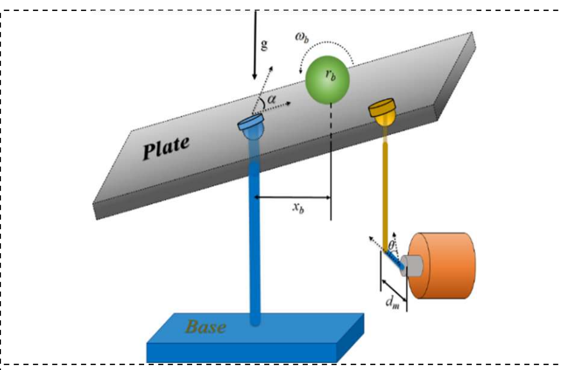

Figure 5. The ball on plate system in terms of potential energy

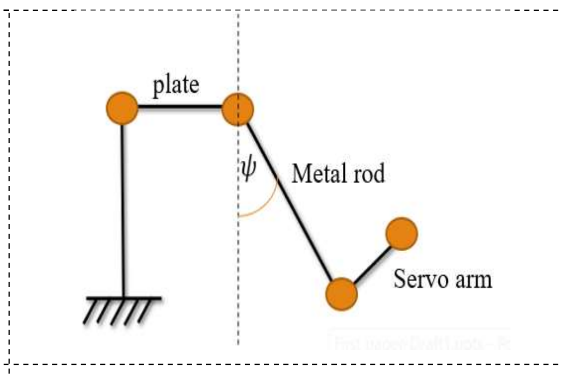

Figure 6. Relation between α and θ in the X-axis

The equation of motion of the ball in case hollow sphere is expressed as follows:

$$\frac{5}{3}\ddot{x_b} + \frac{f_c \dot{x_b}}{m_b} + g\sin\alpha = 0 \quad (11)$$

where f_c is the coefficient of kinetic friction between plate and ball.
Since the plate inclination angle is small ($\pm 14.9°$), it is true to assume that $\sin\alpha \cong \alpha$ to obtain a linearized equation.

The schematic in Fig. 5 also contains an articulated arm that transfer the servo-motors angle θ_x rotation to the plate rotation α_x in the X-axis direction. The prototype has an identical mechanism about the Y-axis direction with another servo motor. By simple calculation it is possible to write the relation between the servo and plate angle as follows:

$$d_m \sin\theta = d_c \sin\alpha \quad (12)$$

Where d_m is the servo-motor arm length and d_c is the distance between the plate central joint and vertical arm joint and $H = \frac{d_c}{d_m}$.

To avoid confusion, the input of the system is the reference angle of the servo θ_r since the angle α is the plate angle. The general second order equation is assumed for the servo- motor dynamics of this system as follows:

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + {\omega_n}^2\theta = {\omega_n}^2\theta_r \quad (13)$$

Where ω_n is the motor's natural frequency, ζ is the motor's damping ratio, and θ_r is the reference servo angle.

This means that the Eq. (12) can be rederived in terms of Eq. (11) and Eq. (13).

The previous relationship between α and θ is obtained assuming that the linking rod is quasi vertical, which means that the angle ψ in Fig. 6 equals zero. This is not a realistic case since a kinematic analysis is taken in considerations. the structure is four bar mechanisms; therefore, the analytical solution is hard to reach. By using the Simscape library in Simulink a geometrical structure can be built the geometrical structure as shown in Fig. 7.the geometrical structure takes an input θ and measure the output α then plot the numerical relationship between α and θ.

After obtaining the analytical solution for the relation between $\alpha\ and\ \theta$ a comparison with the assumption in equation (12) is shown in Fig. 8. the assumption is a very close approximation throughout the operating region. Therefore, the relation $dm\ sin\ \theta\ =\ dc\ sin\ \alpha$ will be adopted in this study. As discussed earlier, the trigonometric terms need to be linearized around the state of equilibrium to come up with a control design, however, we can use a look up table or ML algorithm to map the relation between $\alpha\ and\ \theta$ upon using the real system, so the equation (12) can be written as follow:

$$\theta = H\,\alpha \quad (14)$$

This will give us the equation that will be used to connect the system's input α_r to the system's output x_b.

$$\ddot{\alpha} + 2\zeta\omega_n\dot{\alpha} + {\omega_n}^2\alpha = {\omega_n}^2\alpha_r \quad (15)$$

To obtain $\zeta\ and\ \omega_n$ for the motor, the system parameters identification method is performed. The procedure requires a sensor to measure the angle α resulting from an input α_r. Measuring the angle α requires a Gyroscope to be attached on the central pivoting point of the plate. This measurement method is time consuming and requires a clock module to be connected on the ARDUINO to know the absolute time of the measurement and sync it with the input. A simpler way is to place a smart phone on top of the plate and connect it to MATLAB by WIFI and capture the readings of the phone's gyroscope to get the angle α. To sync the input signal with the measurements, the absolute time of both signals is known and can be matched using simple MATLAB matrix manipulations. After performing the identification process, the system parameters are obtained as shown in table .1.

From the equation (15) and (10), the linear time invariant space model can be expressed as follow:

$$\dot{q}(t) = A.x(t) + B.u(t) \quad (16)$$

$$y(t) = c.q(t) + D.u(t) \quad (17)$$

Where $q = [x_b, \dot{x_b}, \alpha, \dot{\alpha}]^T$ are the system states, the input $u = \alpha_r$ is the reference plate angle, and the system output $y = x_b$.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-5f_c}{7m_b} & \frac{5g}{7} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -{\omega_n}^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_b \\ \dot{x_b} \\ \alpha \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ {\omega_n}^2 \end{bmatrix} \alpha_r \tag{18}$$

$$y = x_b + [0]\,\alpha_r \tag{19}$$

The transfer function can be obtained by laplace transformation as follow:

$$\frac{X_b(s)}{\alpha(s)} = \frac{2507}{s^4+38.5s^3+369.8s^2+111.2s} \tag{20}$$

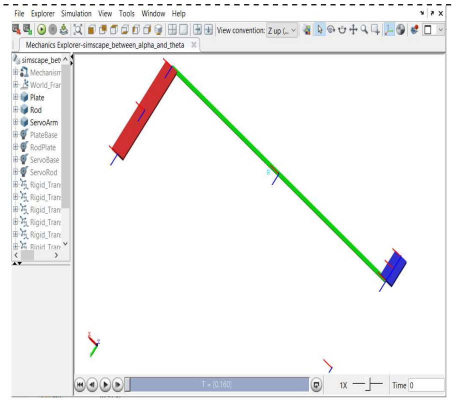

Figure 7. Motion animation of servo motor with plate

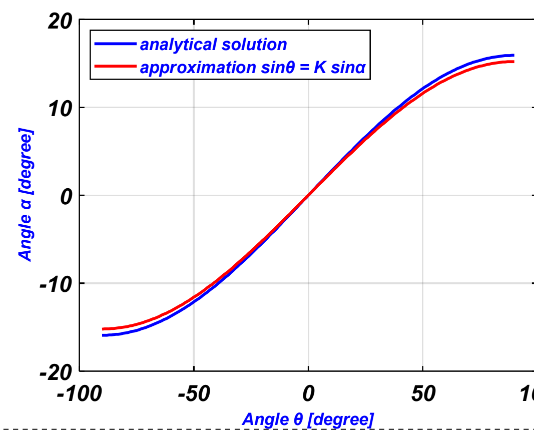

Figure 8. Comparison between the analyitical solution and the vertical rod assumption

3. Pseudo-PD controller.

This section presents a design procedure of Pseudo-PD. Fig. 9 shows the block diagram of Pseudo-PD controller for balancing the ball on the plate. x_b denotes the current position of the ball in x direction. The goal is to balance the ball on the plate in fixed point (i.e., ref=360 pixel) or in trajectory. u and d are the control signal and the external disturbance, respectively. Note that the controller that implemented in this section belongs to X direction and the same controller is implemented to Y direction then the two-controller work in parallel way to control the two servo motors as shown in Fig. 3.

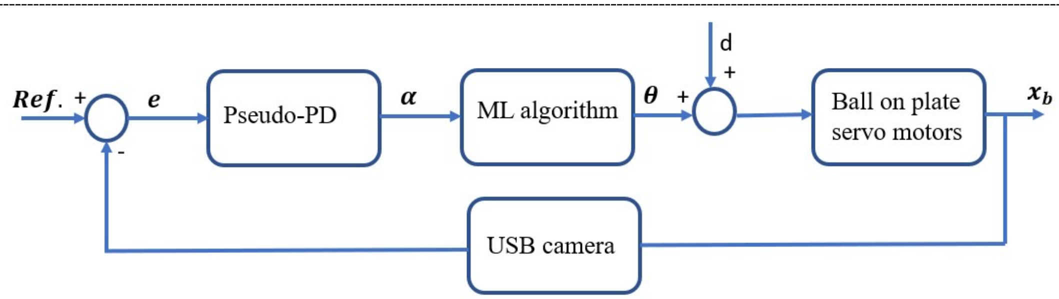

Figure 9. Pseudo-PD controller for ball on the plate system

The Pseudo-PD controller is usually implemented as follows:

$$U(s) = k_p E(s) + K_d \frac{as}{s+a} \qquad (21)$$

The proposed controller for the ball on the plate designed using root locus by the following steps:

(1) the location of open loop poles and zeros is shown in Fig. 10. the system is not BIBO stable and that there are 2 dominant poles that are very close from the real axis that needs to be damped.

(2) By applying the Pseudo-PD controller to the system by adding zero and pole. The response of the system without controller does not give any indication for overshoot or settling time, many controllers are designed with different values of overshoot and settling time.

(3) Determining the requirements of the ball on the plate system.

- Settling time: $\leq 3s$.
- Overshoot: $\leq 5\%$.
- Steady state error: $\leq 4mm$.
- Gain margin: $\geq 10dp$.
- Phase margin: $\geq 44\ deg$.

(4) Calculating the Pseudo-PD parameters after adding the requirements to the root locus as shown in Fig. 10.

(5) The calculated equation from the root locus is defined as follow:

$$c(s) = \frac{9.0817(s+0.8601)}{(s+18.67)} \qquad (22)$$

(6) After calculating the parameters of Pseudo-PD controller from the Eq. (21) that full fill the requirements in step (3) the proposed controller take position error that produced from the difference between the current position taken from the camera and the reference position and calculate the angle α that move the plate in the direction x and the other controller move the plate in the y direction.

(7) The angle α is fed to the ML algorithm that depend on the Eq. (12) to calculate the servo motor angles θ.

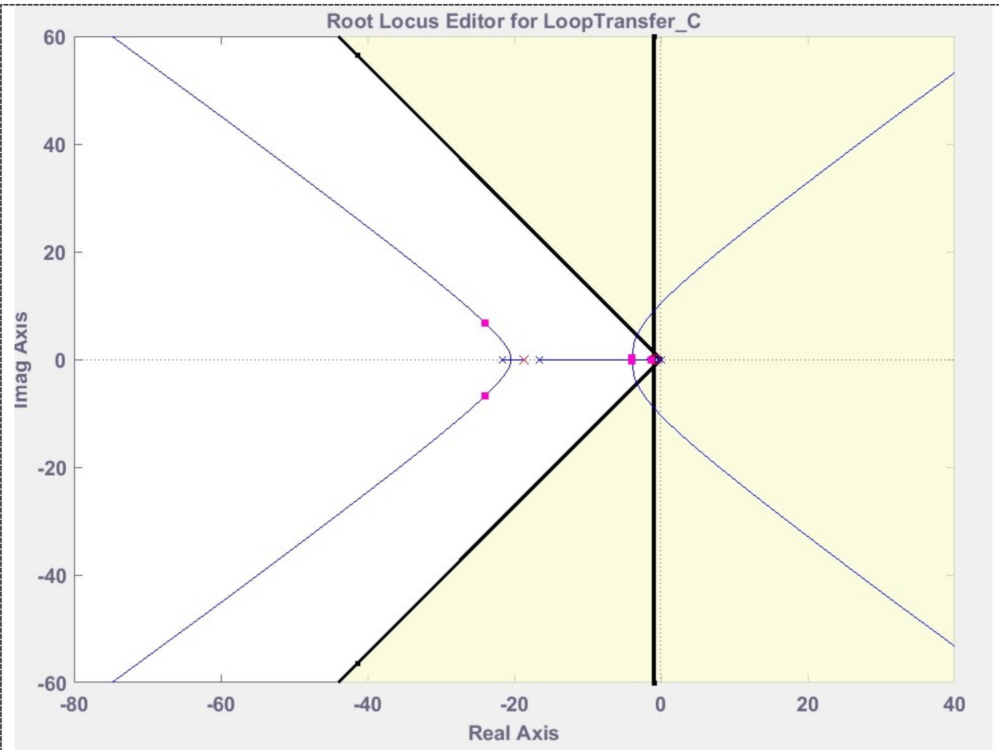

Figure 10. Root locus after adding Pseudo-PD controller to fulfill the requirements

4. Machine learning algorithms.

Machine learning models have been used in many fields such as classification, regression, clustering and used in control systems. the control system dataset can be analyzed with the help of machine learning models. In this paper, different machine learning models for diagnosing and predicting the servo motor angle based on the control system dataset which obtained from the equation (12) and the regression models. The block diagram of the ML model for the predicting the servo motor angle in Fig. 11. First, the datasets are loaded. then these datasets are used as input for the ML regression models. Different models are used in this paper, namely support vector regression (SVR) [14], decision tree regression (DTR) [15], and random forest regression (RFR) models [16]. the three ML models achieved a high accuracy which are 99.58 %, 100 %, and 99.998 % for SVR, DTR and RFR, respectively as shown in Fig. (12), Fig. (13), and Fig. (14). The DTR model achieved the highest performance with accuracy 100% in compared with other models, so it used with the PD controller in section 3.

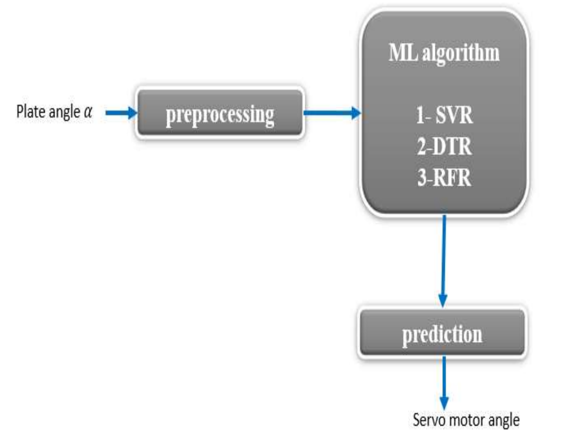

Figure 11. ML block diagram

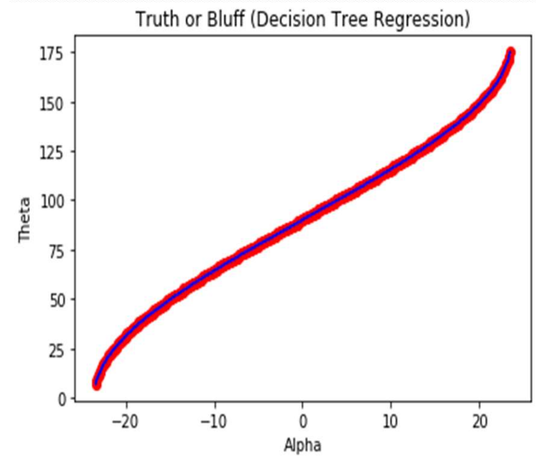

Figure 12. DTR ML algorithm output

Figure 13. RFR ML algorithm output

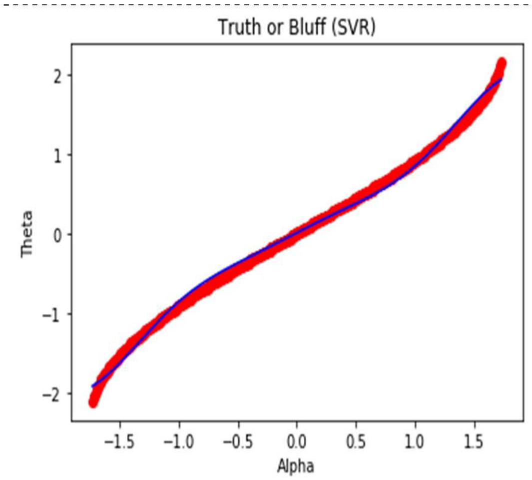

Figure 14. SVR ML algorithm output

5. Simulation Results and Discussion.

The proposed PD controller with ML algorithm improve the settling time to 3 second and overshoot to 2% compared with other papers that used the PD controller only or used advanced controller like fuzzy PD controller or sliding mode controller as shown in Fig. 15.

The comparison of the proposed prediction approach with other published approaches is shown in Table 1.

Table 1. Comparison between the proposed approach and other published approaches.

author	year	*Settling time (sec)*	Overshoot (%)
Faiber et al [11]	2019	4	10
Mustafa Mehedi et al. [13]	2019	4.5	6%
Agung Adiprasetya et al. [12]	2016	> 4	> 5%

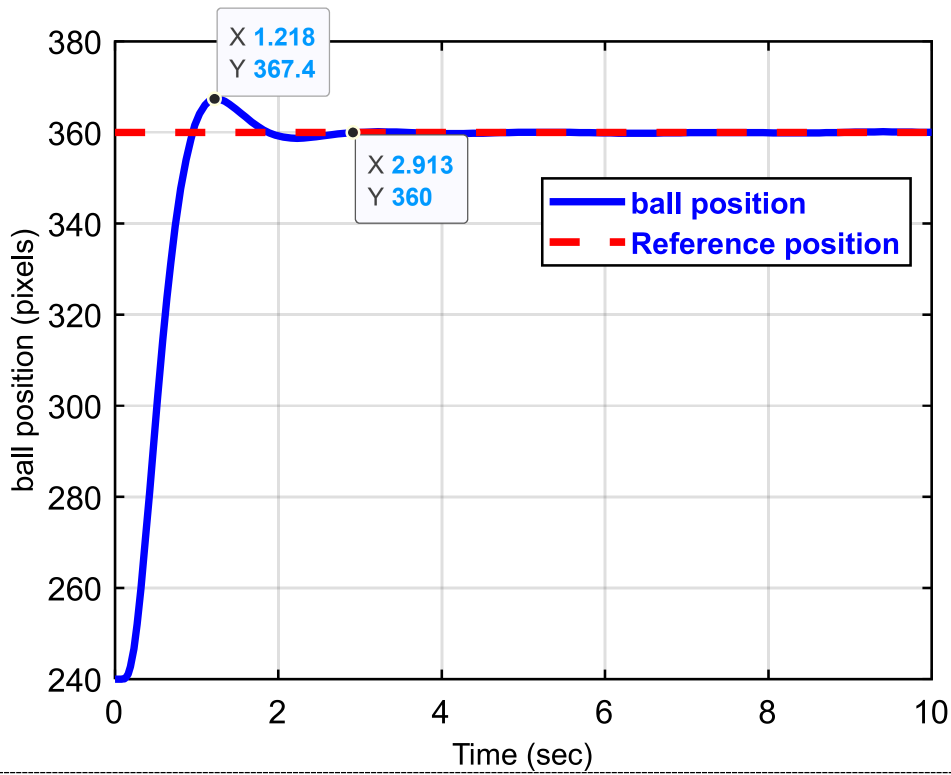

Figure 15. Reference tracking using the tuned PD controller with the Parameters (P = 0.03, D = 0.02, N = 20) with the ML algorithm.

6. Conclusion

This paper presented an efficient approach that depends on ML algorithms that expect the correct angle for servo motors depending on the plate angle. The PD controller calculate the plate angle in x and y direction and give it to the ML algorithm to calculate the servo motor angle. The ML improve the reference tracking of the ball on the plate system (BOPS) by reducing the setting time and overshoot for the system response.

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