



## Exploring a Dichotomy Prime Numbers Divided by a Unique Property

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April 17, 2024

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## Abstract

In this research, we examine a special property of some prime numbers that goes beyond separating primes from odd composites. Rather, this property serves to divide the set of primes into two groups: those having it and those lacking it. Incredibly, these two groups portray an almost equal distribution of primes amongst them. This phenomenon is explored in order to reveal some aspects concerning the basic characteristics and structure of prime numbers. We also provide further insight on what this trait implies within prime number theory through rigorous analysis as well as mathematical inspection which shines a new light on the enigmatic properties of these fundamental mathematical entities.

## 1 Introduction

Mathematicians have always been fascinated by prime numbers which are those integers that are not divisible by other numbers except one, and they are of significant importance in number theory. Even though a lot has been said about their distribution and properties with respect to composite numbers, there is recent focus on a specific feature among the set of prime numbers. Instead of separating prime numbers from odd composites as it would commonly be done so, this attribute goes beyond such simple dichotomy towards the nature of primes themselves into two groups according to some property.

This study explores this special quality that differentiates between primes – those having it and those without. It is interesting to note that during our investigation, we noticed nearly equal proportions of primes in these two categories, suggesting an order beneath the chaos of prime numbers.

There is a much bigger intention at play than just enjoying this situation; there is an attempt to understand the nature and structure of prime numbers. This paper seeks to explore what clue may be drawn from understanding if any on how prime numbers work within this context. By doing so, we seek to illuminate previously unexplored facets of prime numbers, uncovering new insights into their enigmatic nature.

This introduction sets the stage for a journey into the heart of prime number partitioning, where mathematical rigor meets the quest for understanding the fundamental properties of these elemental mathematical entities. Join us as we

delve into the depths of prime number theory, uncovering the secrets that lie within the seemingly simple yet profoundly complex realm of prime numbers. [1][2][3][4][5] [6]

## 2 Exploring the Dichotomy of Prime Numbers Based on a Unique Property

We name the primes  $p$  that have the property that can be written as  $p = t \cdot s - t + 1$ , where  $s$  is the sum of their digits and  $t$  is a non-null positive integer, primes of class I and the primes that haven't it primes of class II.

### Example

- 19 is a prime of class I because it can be written as  $19 = t \cdot s - t + 1$ , where  $s = 10$  and  $t = 2$ ;
- 23 is a prime of class II because it can't be written as  $23 = t \cdot s - t + 1$ , where  $m$  is a non-null positive integer.

## 3 Conjecture

Let  $a$  be the number of primes in class I less than or equal to  $N$ , and  $b$  be the number of primes in class II less than or equal to  $N$ . Given any sufficiently large positive integer  $N$  (probably the condition that  $N > 78$  is sufficient), let  $r$  be the largest value from  $\frac{a}{b}$  and  $\frac{b}{a}$ ; then  $r < 2$ .

### Primes of Hypothesis One

(that have this property) 2, 3, 5, 7, 11, 13, 19, 31, 37, 41, 43, 61, 71, 73, 101, 103, 113, 127, 137, 151, 157, 163, 181, 191, 193, 199, 211, 223, 229, 239, 241, 271, 281 (...). The corresponding values of  $m$ : 1, 1, 1, 1, 10, 4, 2, 10, 4, 10, 7, 10, 10, 8, 100, 34, 16, 14, 34, 7, 13, 18, 20, 19, 16, 11, 70, 37, 19, 40, 30, 28 (...).

### Primes of Hypothesis Two

(that have not this property) 17, 23, 29, 47, 53, 59, 67, 79, 83, 89, 97, 107, 109, 131, 139, 149, 167, 173, 179, 197, 227, 233, 239, 251, 257, 263, 269, 277, 283, 293 (...).

### Investigation

It can be seen that, for  $N = 100$ ,  $r = 14/11$ ; for  $N = 200$ ,  $r = 26/20$ ; for  $N = 300$ ,  $r = 33/30$ .

## Important Note

Based on this characteristic, interesting classes of primes can be created. One such class is made up of the primes  $p$  that have the formula  $p = s^2 - s + 1$ , where  $s$  is the total of their digits. Examples of such primes are 13, 43, 151, 157, and so on.

## 4 Conclusion

To wrap up, our study explores an intriguing aspect of specific prime numbers that goes beyond just identifying primes versus odd composites. Instead, it uncovers how these numbers can be divided into different categories, showing a remarkably even distribution among them. By closely examining and analyzing the mathematics involved, we have highlighted the essential traits and organization of prime numbers. This investigation not only enhances our comprehension of prime number theory but also brings to light fresh perspectives on the mysterious essence of these crucial mathematical elements.

## References

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- [6] Budee U Zaman. The two couriers problem and diverse approaches to division by zero. 2023.