EasyChair Preprint

№ 4482

Paraboctys (part 2)

Charles Kusniec

October 29, 2020

Notes on Number Theory and Discrete Mathematics
Print ISSN 1310–5132, Online ISSN 2367–8275
Vol. XX, 20XX, No. X, XX–XX

Paraboctys (part 2)

Charles Kusniec[1]

[1] e-mail: charleskusniec@icloud.com

Received: 28 Oct 2020 **Revised:** DD Month YYYY **Accepted:** DD Month YYYY

Abstract: This study is a continuation of paraboctys part 1 and continues in paraboctys part 3.
Keywords: Quadratics, Sieve of Primes, elementary number theory.
2010 Mathematics Subject Classification: 11N35, 11N36, 11A05, 11A51.

1 Introduction

This study is a continuation of paraboctys part 1 and continues in paraboctys part 3.

Please, as reference consult Conventions, notations, and abbreviations [2].

In this study, we will explain the basic functioning of paraboctys showing the reason for appearing quadratic sequences.

After justifying the name paraboctys, we will define what is the specific paraboctys in the submarine and destroyer forms.

Then we will define the general paraboctys and its conclusive notation.

Then we will introduce the concept of triangles formed by the Integers in the paraboctys, which we call trianz.

Finally, we will show one application of the trianz's as an example of the separation and classification of the prime numbers.

2 The form of the table and its equations

Realize that as a consequence of the reasoning proposed, the table we arrived at has the following form of construction:

dif d	dif	C	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
		1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
	2	2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
2	4	3	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
2	6	4	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
2	8	5	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61
2	10	6	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
2	12	7	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83
2	14	8	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97
2	16	9	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113
2	18	10	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
2	20	11	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151
2	22	12	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173
2	24	13	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197
2	26	14	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223
2	28	15	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251
2	30	16	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281
2	32	17	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313
2	34	18	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347
2	36	19	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383
2	38	20	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421
2	40	21	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461
2	42	22	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503
2	44	23	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547
2	46	24	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593
2	48	25	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641
2	50	26	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691
2	52	27	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743
2	54	28	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797
2	56	29	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853
2	58	30	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911
2	60	31	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971
2	62	32	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033
2	64	33	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097
2	66	34	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163
2	68	35	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221	1222	1223	1224	1225	1226	1227	1228	1229	1230	1231
2	70	36	1260	1261	1262	1263	1264	1265	1266	1267	1268	1269	1270	1271	1272	1273	1274	1275	1276	1277	1278	1279	1280	1281	1282	1283	1284	1285	1286	1287	1288	1289	1290	1291	1292	1293	1294	1295	1296	1297	1298	1299	1300	1301
2	72	37	1332	1333	1334	1335	1336	1337	1338	1339	1340	1341	1342	1343	1344	1345	1346	1347	1348	1349	1350	1351	1352	1353	1354	1355	1356	1357	1358	1359	1360	1361	1362	1363	1364	1365	1366	1367	1368	1369	1370	1371	1372	1373
2	74	38	1406	1407	1408	1409	1410	1411	1412	1413	1414	1415	1416	1417	1418	1419	1420	1421	1422	1423	1424	1425	1426	1427	1428	1429	1430	1431	1432	1433	1434	1435	1436	1437	1438	1439	1440	1441	1442	1443	1444	1445	1446	1447
2	76	39	1482	1483	1484	1485	1486	1487	1488	1489	1490	1491	1492	1493	1494	1495	1496	1497	1498	1499	1500	1501	1502	1503	1504	1505	1506	1507	1508	1509	1510	1511	1512	1513	1514	1515	1516	1517	1518	1519	1520	1521	1522	1523
2	78	40	1560	1561	1562	1563	1564	1565	1566	1567	1568	1569	1570	1571	1572	1573	1574	1575	1576	1577	1578	1579	1580	1581	1582	1583	1584	1585	1586	1587	1588	1589	1590	1591	1592	1593	1594	1595	1596	1597	1598	1599	1600	1601
2	80	41	1640	1641	1642	1643	1644	1645	1646	1647	1648	1649	1650	1651	1652	1653	1654	1655	1656	1657	1658	1659	1660	1661	1662	1663	1664	1665	1666	1667	1668	1669	1670	1671	1672	1673	1674	1675	1676	1677	1678	1679	1680	1681
2	82	42	1722	1723	1724	1725	1726	1727	1728	1729	1730	1731	1732	1733	1734	1735	1736	1737	1738	1739	1740	1741	1742	1743	1744	1745	1746	1747	1748	1749	1750	1751	1752	1753	1754	1755	1756	1757	1758	1759	1760	1761	1762	1763

Figure 1: Table with the rows growing downwards in the opposite direction of the Y-axis and the columns growing right in the direction of the X-axis.

- In the 1st row are the Natural numbers or $(\mathbb{N} + 0)$.
- In the 2nd row are the Natural numbers from the 1st row added by 2 or $(\mathbb{N} + 2)$.
- In the 3rd row are the Natural numbers from the 1st row added by 6 or $(\mathbb{N} + 6)$.
- In the 4th row are the Natural numbers from the 1st row added by 12 or $(\mathbb{N} + 12)$.
- In the 5th row are the Natural numbers from the 1st row added by 20 or $(\mathbb{N} + 20)$.
- And so on…

Now, to study this table, it will be easier to have a quick access mechanism to denote any element and/or sequences of the table.

The easiest way to map this table is to use the Cartesian coordinate system. If we invert the direction of the rows, we can map it using the usual two dimensions (x, y) coordinates.

Then, we can refer to the position of each element as we do in the XY-plane. Let's see:

dif dif	dif	C	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
2	82	42	1722	1723	1724	1725	1726	1727	1728	1729	1730	1731	1732	1733	1734	1735	1736	1737	1738	1739	1740	1741	1742	1743	1744	1745
2	80	41	1640	1641	1642	1643	1644	1645	1646	1647	1648	1649	1650	1651	1652	1653	1654	1655	1656	1657	1658	1659	1660	1661	1662	1663
2	78	40	1560	1561	1562	1563	1564	1565	1566	1567	1568	1569	1570	1571	1572	1573	1574	1575	1576	1577	1578	1579	1580	1581	1582	1583
2	76	39	1482	1483	1484	1485	1486	1487	1488	1489	1490	1491	1492	1493	1494	1495	1496	1497	1498	1499	1500	1501	1502	1503	1504	1505
2	74	38	1406	1407	1408	1409	1410	1411	1412	1413	1414	1415	1416	1417	1418	1419	1420	1421	1422	1423	1424	1425	1426	1427	1428	1429
2	72	37	1332	1333	1334	1335	1336	1337	1338	1339	1340	1341	1342	1343	1344	1345	1346	1347	1348	1349	1350	1351	1352	1353	1354	1355
2	70	36	1260	1261	1262	1263	1264	1265	1266	1267	1268	1269	1270	1271	1272	1273	1274	1275	1276	1277	1278	1279	1280	1281	1282	1283
2	68	35	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213
2	66	34	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145
2	64	33	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079
2	62	32	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015
2	60	31	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953
2	58	30	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893
2	56	29	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835
2	54	28	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779
2	52	27	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725
2	50	26	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673
2	48	25	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623
2	46	24	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575
2	44	23	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529
2	42	22	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485
2	40	21	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443
2	38	20	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403
2	36	19	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365
2	34	18	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329
2	32	17	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295
2	30	16	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263
2	28	15	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233
2	26	14	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205
2	24	13	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179
2	22	12	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155
2	20	11	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133
2	18	10	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113
2	16	9	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
2	14	8	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
2	12	7	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65
2	10	6	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
2	8	5	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
2	6	4	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
2	4	3	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
2	2	2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	0	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

Figure 1. Table with the rows growing up along the Y-axis and the columns growing right along the X-axis.

From now on, all along with these studies, we will use

- Y-axis with variable y as the index value of the rows of the table, and
- X-axis with variable x or C as the index value of the columns of the table.

Then we can call each element of our table as being a function $Y[C,y]$, where the variable C is the column value and the coordinate y is the row value.

So, in algebraic terms, we can write:

$$Y[C,y] = Y[(C-1),y] + 1$$
$$Y[C,y] = Y[C,(y-1)] + dif[y]$$

Where $dif[y]$ is the sequence from our reasoning $(0,2,4,6,8,10,\dots)$ in the function of index y in Y-axis, or:

$$dif[y] = 2(y-1)$$

So, we can write

$$Y[C,y] = Y[(C-1),y] + 1$$
$$Y[C,y] = Y[C,(y-1)] + 2(y-1)$$

In the horizontal rows, there is no mystery. The form of all the horizontal rows is the consecutive form of the Integer numbers.

But in the verticals and some diagonals, there are very visible Prime numbers sequences. So, let's try to understand what happens with the elements in the vertical columns when we vary index y.

When $y = 1$, then $Y[C, 1] = C$. Or, when $y = 1$, then $Y[C, 1]$ is the value of the column C. So,

$$Y[C, 1] = C$$

From our system

$$Y[C, y] = Y[(C - 1), y] + 1$$
$$Y[C, y] = Y[C, (y - 1)] + 2(y - 1)$$

We have

$$Y[C, 2] = Y[C, 1] + 2 * 1$$
$$Y[C, 3] = Y[C, 2] + 2 * 2$$
$$Y[C, 4] = Y[C, 3] + 2 * 3$$
$$Y[C, 5] = Y[C, 4] + 2 * 4$$
$$Y[C, 6] = Y[C, 5] + 2 * 5$$
...

Or

$$Y[C, 1] = Y[C, 1]$$
$$Y[C, 2] = Y[C, 1] + 2 * 1$$
$$Y[C, 3] = Y[C, 1] + 2 * 1 + 2 * 2$$
$$Y[C, 4] = Y[C, 1] + 2 * 1 + 2 * 2 + 2 * 3$$
$$Y[C, 5] = Y[C, 1] + 2 * 1 + 2 * 2 + 2 * 3 + 2 * 4$$
$$Y[C, 6] = Y[C, 1] + 2 * 1 + 2 * 2 + 2 * 3 + 2 * 4 + 2 * 5$$
...

Because we started in $Y[C, 1] = C$, we can write:

$$Y[C, 1] = C$$
$$Y[C, 2] = C + 2 * 1$$
$$Y[C, 3] = C + 2 * 1 + 2 * 2$$
$$Y[C, 4] = C + 2 * 1 + 2 * 2 + 2 * 3$$
$$Y[C, 5] = C + 2 * 1 + 2 * 2 + 2 * 3 + 2 * 4$$
$$Y[C, 6] = C + 2 * 1 + 2 * 2 + 2 * 3 + 2 * 4 + 2 * 5$$
...

Generically writing:

$$Y[C, y] = C + 2 * 1 + 2 * 2 + 2 * 3 + 2 * 4 + 2 * 5 + \cdots + 2(y - 1)$$

So,

$$Y[C, y] = C + 2\big(1 + 2 + 3 + 4 + 5 + \cdots + (y - 1)\big)$$

Or,

$$Y[C, y] = C + 2\left(\frac{y(y - 1)}{2}\right)$$

Then,

$$Y[C, y] = y^2 - y + C$$
$$Y[C, y] = Oblong\ number + C = A002378 + C$$

This means we have sequences in verticals of the form of a 2nd-degree polynomial. We call these sequences quadratics or parabolas. So, the equations give the sequences in each column:

$$Y[0, y] = y^2 - y$$
$$Y[1, y] = y^2 - y + 1$$
$$Y[2, y] = y^2 - y + 2$$
$$Y[3, y] = y^2 - y + 3$$
$$Y[4, y] = y^2 - y + 4$$
$$Y[5, y] = y^2 - y + 5$$
...

Note: our vertical sequences can be denoted as a sequence of elements of the form of a function just as $Y[y]$.

3 Extending the table to the negative indexes

Realize that nothing prevents us from using negative columns C and/or negative indexes y values on the equation of the verticals.

So, using $Y[y] = y^2 - y + C$, we can extend our table to cover all Integer values for columns and rows.

See what we get:

a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
21	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441
20	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
19	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
18	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
17	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
16	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
15	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
14	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
13	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
12	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
11	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
10	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
9	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
8	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
7	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
4	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
3	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
2	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-1	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
-2	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
-3	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
-4	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
-5	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
-6	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
-7	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
-8	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
-9	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
-10	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
-11	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
-12	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
-13	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
-14	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
-15	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
-16	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
-17	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
-18	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
-19	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
-20	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441

Figure 1. Table resulting from our reasoning.

What G'd invented here to be discovered by us?

4 Why appeared quadratics on the verticals?

The method of finite differences can explain and justify the 2nd-degree polynomial equation $Y[y] = y^2 - y + C$ that we have just arrived in the form to express the vertical sequences formed in the table.

The 2nd-degree polynomial equation that we have just arrived in the form $Y[y] = y^2 - y + C$ to express the vertical sequences formed in the table can be explained and justified by the method of finite differences.

This is because the table we have arrived at results from the reasoning that has the following algorithm:

- from the row $y = 1$, we added 2 to get the row $y = 2$.
- from the row $y = 2$, we added 4 to get the row $y = 3$.
- from the row $y = 3$, we added 6 to get the row $y = 4$.
- from the row $y = 4$, we added 8 to get the row $y = 5$.
- And so on…

This means that

- the result of the difference between the elements of row $y = 2$ minus the elements of row $y = 1$, is $dif = 2$.
- the result of the difference between the elements of row $y = 3$ minus the elements of row $y = 2$, is $dif = 4$.
- the result of the difference between the elements of row $y = 4$ minus the elements of row $y = 3$, is $dif = 6$.
- the result of the difference between the elements of row $y = 5$ minus the elements of row $y = 4$, is $dif = 8$.
- And so on.

This also means that

- the result of the difference between (elements of row $y = 3$ minus elements of row $y = 2$) minus the (elements of row $y = 2$ minus the elements of row $y = 1$), is $difdif = 4 - 2 = 2$.
- the result of the difference between (elements of row $y = 4$ minus elements of row $y = 3$) minus the (elements of row $y = 3$ minus the elements of row $y = 2$), is $difdif = 6 - 4 = 2$.
- the result of the difference between the (elements of row $y = 5$ minus elements of row $y = 4$) minus (elements of row $y = 4$ minus the elements of row $y = 3$), is $difdif = 8 - 6 = 2$.
- And so on.

If we now calculate a new level of differences, we get $difdifdif = 0$. When we get to the Zero difference level, it means that there will be no new levels and so we stop the process.

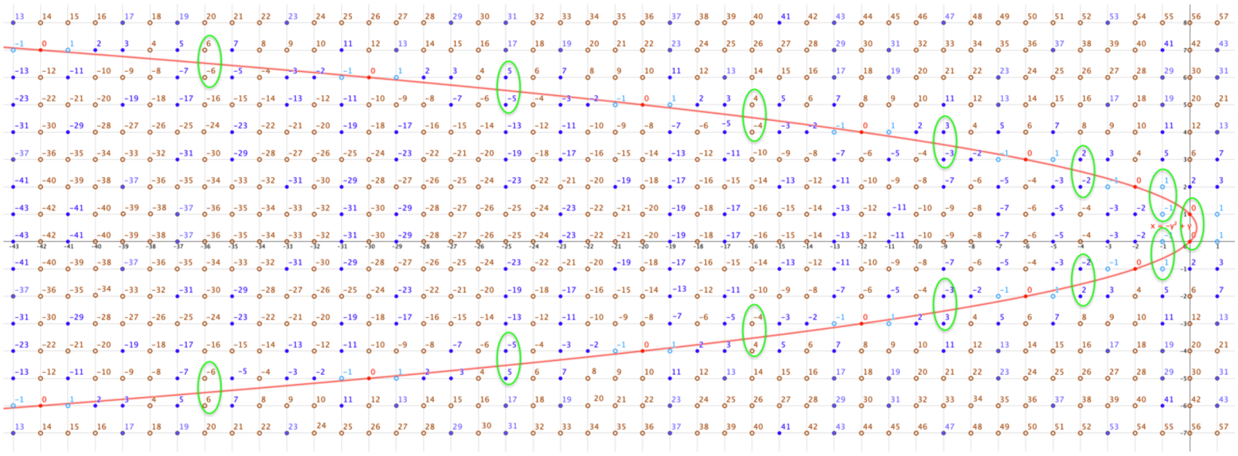
So, since we have only 2 different levels of the differences, the method of finite differences assures us we are dealing with a 2nd-degree polynomial.

And yet, as $difdif = 2$ then the method of finite differences also ensures that the coefficient of the 2nd-degree is $a = \frac{iterations\ of\ dif}{n!} = \frac{difdif}{2!} = \frac{2}{2} = 1$. See more details in the study "The simplest polynomial equations, the inflection point, the recurrence equations up to degree 6, and the method of finite differences".

4.1 Any even number results from a difference between 2 Primes

Because the table we have arrived at has the following algorithm:

- from the row $y = 1$, we added 2 to get the row $y = 2$.
- from the row $y = 2$, we added 4 to get the row $y = 3$.
- from the row $y = 3$, we added 6 to get the row $y = 4$.
- from the row $y = 4$, we added 8 to get the row $y = 5$.
- And so on…



Row\Col	-n^2+2
n+1	n+2
n	-n+2
dif	2n
-n+1	-n+2
-n	n+2
dif	-2n

Row\Col	-23
6	7
5	-3
dif	10
-4	-3
-5	7
dif	-10

Row\Col	-n^2+1
n+1	n+1
n	-n+1
dif	2n
-n+1	-n+1
-n	n+1
dif	-2n

Row\Col	-15
5	5
4	-3
dif	8
-3	-3
-4	5
dif	-8

Row\Col	-35
7	7
6	-5
dif	12
-5	-5
-6	7
dif	-12

n		1		2		3		4		5		6		7	
Row\Col	-n^2	Row\Col	-1	Row\Col	-4	Row\Col	-9	Row\Col	-16	Row\Col	-25	Row\Col	-36	Row\Col	-49
n+1	n	2	1	3	2	4	3	5	4	6	5	7	6	8	7
n	-n	1	-1	2	-2	3	-3	4	-4	5	-5	6	-6	7	-7
dif	2n	dif	2	dif	4	dif	6	dif	8	dif	10	dif	12	dif	14
-n+1	-n	0	-1	-1	-2	-2	-3	-3	-4	-4	-5	-5	-6	-6	-7
-n	n	-1	1	-2	2	-3	3	-4	4	-5	5	-6	6	-7	7
dif	-2n	dif	-2	dif	-4	dif	-6	dif	-8	dif	-10	dif	-12	dif	-14

Row\Col	-n^2-1
n+1	n-1
n	-n-1
dif	2n
-n+1	-n-1
-n	n-1
dif	-2n

Row\Col	-17
5	3
4	-5
dif	8
-3	-5
-4	3
dif	-8

Row\Col	-37
7	5
6	-7
dif	12
-5	-7
-6	5
dif	-12

Row\Col	-n^2-2
n+1	n-2
n	-n-2
dif	2n
-n+1	-n-2
-n	n-2
dif	-2n

Row\Col	-27
6	3
5	-7
dif	10
-4	-7
-5	3
dif	-10

Therefore, if we prove that for all consecutive row pairs in our table, there will always be at least one vertical sequence of 2 Positive Primes in any column of the table, then we can prove that *"any even number results from a difference between 2 Primes"*.

See that in Figure 1 above, we have the following negative columns with the sequences of two Primes whose differences result in Even numbers:

4.2 Any even number results from a sum of 2 Primes

Thus, refining our reasoning a little, if we sophisticate the proof showing that for all pairs of consecutive rows there is at least one column with a sequence of two primes in the vertical where the minuend is a positive prime and the subtracting is a negative prime, then we prove that *"any even number results from a sum of 2 Primes"*.

See that in Figure 1 above, we have the following negative columns with the sequences of two Primes whose differences result in Even numbers:

5 Origin of the name paraboctys

Continuing with the exploration with our extended table, it draws our attention to the diagonals with the sequence of the perfect square numbers in yellow.

Note that each perfect square number diagonal delimits a triangle with the shape of a Tetractys.

Because of the inherent symmetry of the vertical quadratic sequences, all triangles of Tetractys are dual.

In the table, there is an infinity of Tetractys type triangles. The easiest to see is right in the center.

As shown in Figure 1 below, the 2 yellow diagonals in angles of ±45˚ delimit the central dual-Tetractys extended:

a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
21	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441
20	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
19	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
18	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
17	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
16	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
15	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
14	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
13	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
12	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
11	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
10	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
9	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
8	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
7	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
4	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
3	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
2	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-1	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
-2	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
-3	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
-4	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
-5	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
-6	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
-7	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
-8	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
-9	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
-10	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
-11	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
-12	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
-13	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
-14	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
-15	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
-16	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
-17	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
-18	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
-19	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
-20	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441

Figure 1. The dual-Tetractys extended in the center.

The other infinitely many dual-Tetractys extended triangles spread in different inclinations and offset all along with the table to infinite.

a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
21	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441
20	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
19	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
18	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
17	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
16	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
15	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
14	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
13	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
12	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
11	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
10	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
9	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
8	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
7	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
4	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
3	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
2	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-1	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
-2	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
-3	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
-4	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
-5	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
-6	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
-7	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
-8	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
-9	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
-10	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
-11	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
-12	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
-13	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
-14	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
-15	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
-16	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
-17	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
-18	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
-19	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
-20	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441

Figure 1. The dual-Tetractys extended next to the center.

a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
21	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441
20	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
19	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
18	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
17	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
16	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
15	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
14	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
13	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
12	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
11	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
10	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
9	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
8	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
7	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
4	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
3	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
2	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-1	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
-2	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
-3	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
-4	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
-5	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
-6	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
-7	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
-8	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
-9	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
-10	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
-11	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153
-12	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
-13	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203
-14	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
-15	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261
-16	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293
-17	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327
-18	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363
-19	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
-20	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441

Figure 1. More dual-Tetractys extended. And so on….

When we study this table, we notice many things.

5.1 Symmetry that reminds The Kybalion

The symmetry of the table is because of the symmetry that exists in the parabolas. It is the symmetry of the parabolas that produces here the symmetrical quadratic sequences.

In The Kybalion, it is written: *"**Principle of Polarity**: The Principle of Polarity embodies the idea that everything is dual, everything has two poles, and everything has its opposite. All manifested things have two sides, two aspects, or two poles. Everything "is" and "isn't" at the same time, all truths are but half-truths and every truth are half false, there are two sides to everything, opposites are identical in nature, yet different in degree, extremes meet, and all paradoxes may be reconciled."*

5.2 Parabola that reminds the Hyperbola

We define a parabola as a set of points in a plane equidistant from a straight line or directrix and focus.

We can define the hyperbola as the difference of distances between a set of points, which are present in a plane to two fixed points, is a positive constant.

5.3 Quadratics that reminds the Inverse Square Law

In science, an Inverse Square Law is any scientific law stating that a specified physical quantity is inversely proportional to the square of the distance from the source of that physical quantity. We can understand the fundamental cause for this as geometric dilution corresponding to point-source radiation into three-dimensional space.

5.4 Tetractys that reminds Pythagoras, and past studies.

The Tetractys is a triangular figure consisting of ten points arranged in four rows: one, two, three, and four points in each row, which is the geometrical representation of the fourth triangular number.

The Tetractys triangle form was already extended by Athanasius Kircher - Mundus subterraneous (1664).

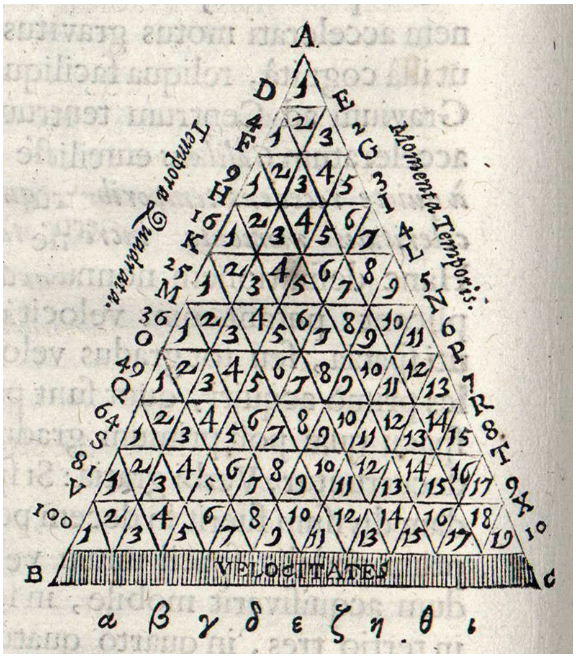


Figure 1. The Tetractys extended triangle by Athanasius Kircher - Mundus subterraneous (1664).

We mention the Tetractys triangle in OEIS as a solution to a puzzle from Copyright of the Prime Pages' Prime Curios! is held by its Managing Editors: G. L. Honaker, Jr. and Chris Caldwell. The solution provided by OEIS is on A135948.

Also, there is a mention at A000217 Triangular numbers saying a(4): points in Pythagorean Tetractys.

Also, the Tetractys extended triangle was studied by Laurence Monroe Klauber in 1932 to sift Prime sequences.

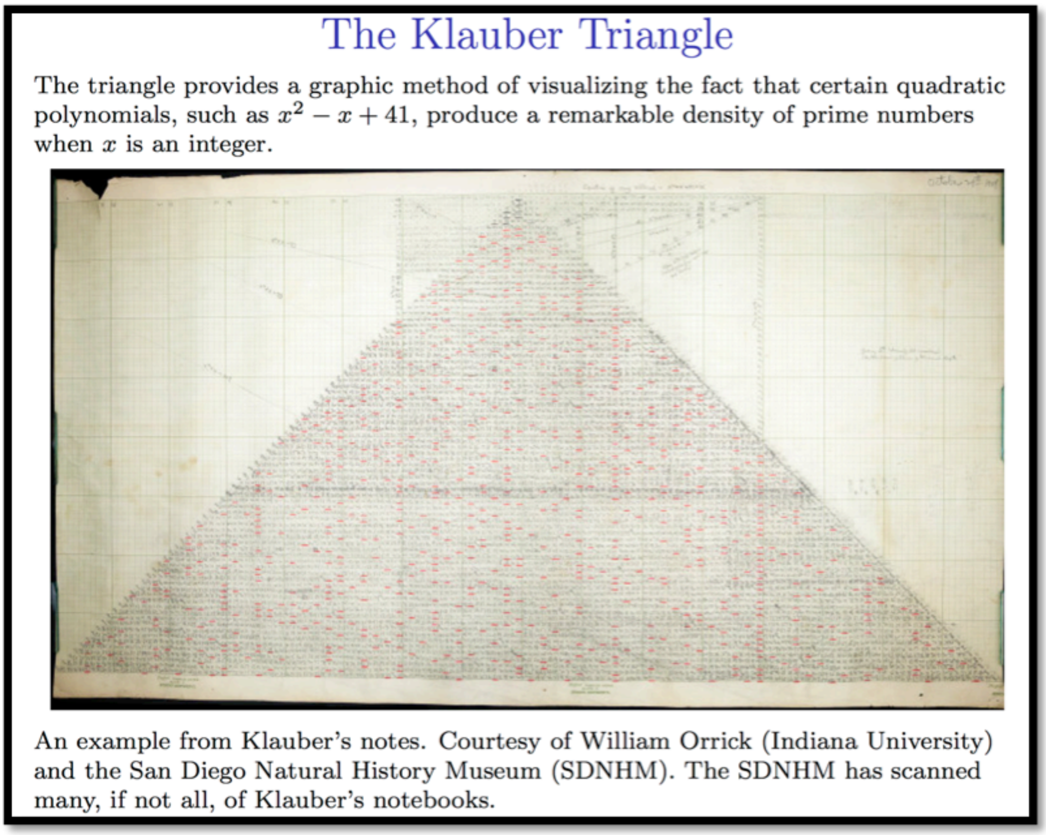


Figure 1. The Klauber Triangle.

The Klauber Triangle is the oldest study we have found close to our studies. We can understand that the table we have arrived at extends the Klauber Triangle.

See that at the top of the photo he makes some essays to extend his triangle. The photo is not very clear, but he probably tried to extend it to both the positive and the negative side. It would interest to see more clearly what exactly he wrote. Probably, if in his time there were computers like nowadays, certainly he would have arrived in our complete table.

There is also a past study of the Klauber triangle in the link OEIS study. We cannot detect who the real author of this photo is.

5.5 Final soup

Because this table represents infinitely many PARABOlas with infinitely many dual-tetraCTYS extended triangles, we gave the name PARABOCTYS for this table.

6 Definition of the specific paraboctys:

The extended table that we have reached allows us to define the specific paraboctys with the following algorithm:

6.1 The three initial consecutive elements

Each column is a 2nd-degree polynomial equation of the form

$$Y[y] = ay^2 + by + c$$

The 3 consecutive elements of a vertical sequence $[Y[-1], Y[0], Y[1]] = [x_1, x_2, x_3]$ determine the coefficients. Each vertical quadratic equation is

$$[x_1, x_2, x_3] \equiv Y[y] = \left(\frac{x_1 - 2x_2 + x_3}{2}\right) y^2 + \left(\frac{x_3 - x_1}{2}\right) y + x_2$$

Let's define the three initial consecutive elements as $[x_1, x_2, x_3] = [g, 0, i]$. Then, the central column of the specific paraboctys has the following equation of 2nd-degree:

$$[g, 0, \mathrm{i}] \equiv Y[y] = \left(\frac{\mathrm{g} + \mathrm{i}}{2}\right) y^2 + \left(\frac{\mathrm{i} - \mathrm{g}}{2}\right) y$$

Then, the central column of the specific paraboctys has the following equation of 2nd-degree:

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a						(g+i)/2				
b						(i-g)/2				
c						0				
	4					6g+10i				
	3					3g+6i				
	2					g+3i				
Y[1]=x_3	1					i				
Y[0]=x_2	0	-4	-3	-2	-1	0	1	2	3	4
Y[-1]=x_1	-1					g				
	-2					3g+i				
	-3					6g+3i				
	-4					10g+6i				

Figure 1. The central column of any specific paraboctys.

See how the triangular numbers A000217 {0,1,3,6,10,15,21,28,36,45,55,66, ...} are present in the coefficients of the elements equations that form the central column.

From now on, always keep in mind that:

1. $sequence\ of\ oblong\ numbers = sequence\ of\ triangular\ numbers(n) + sequence\ of\ triangular\ numbers(n)$
2. $sequence\ of\ square\ numbers = sequence\ of\ triangular\ numbers(n) + sequence\ of\ triangular\ numbers(n-1)$

6.2 The specific paraboctys

Thus, the specific paraboctys can be obtained by adding one unit to each right column in each of the center column elements and subtracting one unit to each left column in each of the center column elements. See what we get:

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		(g+i)/2	(g+i)/2	(g+i)/2	(g+i)/2	(g+i)/2	(g+i)/2	(g+i)/2	(g+i)/2	(g+i)/2
b		(i-g)/2	(i-g)/2	(i-g)/2	(i-g)/2	(i-g)/2	(i-g)/2	(i-g)/2	(i-g)/2	(i-g)/2
c		-4	-3	-2	-1	0	1	2	3	4
	4	(6g+10i)-4	(6g+10i)-3	(6g+10i)-2	(6g+10i)-1	6g+10i	(6g+10i)+1	(6g+10i)+2	(6g+10i)+3	(6g+10i)+4
	3	(3g+6i)-4	(3g+6i)-3	(3g+6i)-2	(3g+6i)-1	3g+6i	(3g+6i)+1	(3g+6i)+2	(3g+6i)+3	(3g+6i)+4
	2	(g+3i)-4	(g+3i)-3	(g+3i)-2	(g+3i)-1	g+3i	(g+3i)+1	(g+3i)+2	(g+3i)+3	(g+3i)+4
Y[1]=x_3	1	i-4	i-3	i-2	i-1	i	i+1	i+2	i+3	i+4
Y[0]=x_2	0	-4	-3	-2	-1	0	1	2	3	4
Y[-1]=x_1	-1	g-4	g-3	g-2	g-1	g	g+1	g+2	g+3	g+4
	-2	(3g+i)-4	(3g+i)-3	(3g+i)-2	(3g+i)-1	3g+i	(3g+i)+1	(3g+i)+2	(3g+i)+3	(3g+i)+4
	-3	(6g+3i)-4	(6g+3i)-3	(6g+3i)-2	(6g+3i)-1	6g+3i	(6g+3i)+1	(6g+3i)+2	(6g+3i)+3	(6g+3i)+4
	-4	(10g+6i)-4	(10g+6i)-3	(10g+6i)-2	(10g+6i)-1	10g+6i	(10g+6i)+1	(10g+6i)+2	(10g+6i)+3	(10g+6i)+4

Figure 1. The specific paraboctys. To the right, we add one unit, to the left we decrease one unit.

Note that, whatever the specific paraboctys defined by integers g, i, the coefficients a, and b never change in vertical quadratics. Check in the quadratics general formula why. Interesting, isn't it?

6.3 The submarine specific paraboctys

In cases where $g = i \neq 0$, all vertical columns will have coefficients $a = g$ and $b = 0$.

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		g	g	g	g	g	g	g	g	g
b		0	0	0	0	0	0	0	0	0
c		-4	-3	-2	-1	0	1	2	3	4
	4	16g-4	16g-3	16g-2	16g-1	16g	16g+1	16g+2	16g+3	16g+4
	3	9g-4	9g-3	9g-2	9g-1	9g	9g+1	9g+2	9g+3	9g+4
	2	4g-4	4g-3	4g-2	4g-1	4g	4g+1	4g+2	4g+3	4g+4
Y[1]=x_3	1	g-4	g-3	g-2	g-1	g	g+1	g+2	g+3	g+4
Y[0]=x_2	0	4	3	2	1	0	1	2	3	4
Y[-1]=x_1	-1	g-4	g-3	g-2	g-1	g	g+1	g+2	g+3	g+4
	-2	4g-4	4g-3	4g-2	4g-1	4g	4g+1	4g+2	4g+3	4g+4
	-3	9g-4	9g-3	9g-2	9g-1	9g	9g+1	9g+2	9g+3	9g+4
	-4	16g-4	16g-3	16g-2	16g-1	16g	16g+1	16g+2	16g+3	16g+4

Figure 1. The submarine specific paraboctys.

Now, see how the Square numbers sequence A000290 $\{0,1,4,9,16,25,36,49,64,81,100,\dots\}$ is present in the coefficients of the elements equations that form the central column.

This happens because

$$\mathrm{A000217}(n-1) = \{\dots,0,0,1,3,6,10,15,21,28,36,45,\dots\}$$

$$+$$

$$A000217(n) = \{\dots,0,1,3,6,10,15,21,28,36,45,55,\dots\}$$

$$-----------------------$$

$$A000290(n) = \{0,1,4,9,16,25,36,49,64,81,100,\dots\}$$

Or we can say:

$$Square(n) = Triangular(n) + Triangular(n-1)$$

$$A000290(n) = A000217(n) + A000217(n-1)$$

See, the Offset theory [3] allows us to write this sum of triangular numbers sequences by infinite algebraic forms. For example:

$$\dots$$

$$y^2 + 0y + 0 = (0.5y^2 - 0.5y + 0) + (0.5y^2 + 0.5y + 0)$$

$$y^2 + 2y + 1 = (0.5y^2 + 0.5y + 0) + (0.5y^2 + 1.5y + 1)$$

$$y^2 + 4y + 4 = (0.5y^2 + 1.5y + 1) + (0.5y^2 + 2.5y + 3)$$

$$y^2 + 6y + 9 = (0.5y^2 + 2.5y + 3) + (0.5y^2 + 3.5y + 6)$$

$$\dots$$

That's why it's important to always think of the reference in offset f=0.

6.4 The destroyer specific paraboctys

In cases where $g \neq i = 0$, all vertical columns will have coefficients $a = -b = \frac{g}{2}$.

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		g/2	g/2	g/2	g/2	g/2	g/2	g/2	g/2	g/2
b		-g/2	-g/2	-g/2	-g/2	-g/2	-g/2	-g/2	-g/2	-g/2
c		-4	-3	-2	-1	0	1	2	3	4
	4	6g-4	6g-3	6g-2	6g-1	6g	6g+1	6g+2	6g+3	6g+4
	3	3g-4	3g-3	3g-2	3g-1	3g	3g+1	3g+2	3g+3	3g+4
	2	g-4	g-3	g-2	g-1	g	g+1	g+2	g+3	g+4
Y[1]=x_3	1	4	3	2	1	0	1	2	3	4
Y[0]=x_2	0	4	3	2	1	0	1	2	3	4
Y[-1]=x_1	-1	g-4	g-3	g-2	g-1	g	g+1	g+2	g+3	g+4
	-2	3g-4	3g-3	3g-2	3g-1	3g	3g+1	3g+2	3g+3	3g+4
	-3	6g-4	6g-3	6g-2	6g-1	6g	6g+1	6g+2	6g+3	6g+4
	-4	10g-4	10g-3	10g-2	10g-1	10g	10g+1	10g+2	10g+3	10g+4

Figure 1. The destroyer specific paraboctys.

If we want to use these specific paraboctys to find the prime number sequences on verticals and diagonals, then we need to have two basic things. First, all the elements forming the quadratic sequences must be Integers. Second, all the coefficients of the quadratic equations must also be Integers.

If the elements of the quadratic sequence are Integers, but the coefficients of the equation are not (they may be $\frac{Odd}{2}$), there will be no prime number sequence because there will be odd and even numbers interleaved by following the interleaving present in the triangular numbers.

With this "prime goal" in mind, the value of g cannot be odd. So, let's do $g = 2a$ and see what we get:

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		a	a	a	a	a	a	a	a	a
b		−a	−a	−a	−a	−a	−a	−a	−a	−a
c		-4	-3	-2	-1	0	1	2	3	4
	4	12a-4	12a-3	12a-2	12a-1	12a	12a+1	12a+2	12a+3	12a+4
	3	6a-4	6a-3	6a-2	6a-1	6a	6a+1	6a+2	6a+3	6a+4
	2	2a-4	2a-3	2a-2	2a-1	2a	2a+1	2a+2	2a+3	2a+4
Y[1]=x_3	1	4	3	2	1	0	1	2	3	4
Y[0]=x_2	0	4	3	2	1	0	1	2	3	4
Y[-1]=x_1	-1	2a-4	2a-3	2a-2	2a-1	2a	2a+1	2a+2	2a+3	2a+4
	-2	6a-4	6a-3	6a-2	6a-1	6a	6a+1	6a+2	6a+3	6a+4
	-3	12a-4	12a-3	12a-2	12a-1	12a	12a+1	12a+2	12a+3	12a+4
	-4	20a-4	20a-3	20a-2	20a-1	20a	20a+1	20a+2	20a+3	20a+4

Figure 1. The destroyer specific paraboctys.

Now, see how the Oblong numbers A002378 $\{0,2,6,12,20,30,42,56,72,90,110,\ldots\}$ are present in the coefficients of the elements equations that form the central column.

This happens because

$$A000217(n) = \{\ldots,0,1,3,6,10,15,21,28,36,45,55,\ldots\}$$
$$+$$
$$A000217(n) = \{\ldots,0,1,3,6,10,15,21,28,36,45,55,\ldots\}$$
$$-------------------------$$
$$A002378(n) = \{0,2,6,12,20,30,42,56,72,90,110,\ldots\}$$

Or we can say:

$$Oblong(n) = Triangular(n) + Triangular(n)$$
$$A002378(n) = A000217(n) + A000217(n)$$

Also here, the "Offset theory" study [3] allows us to write this sum of triangular numbers sequences by infinite algebraic forms. This also shows us why the offset $f = 0$ is very important as a reference in infinite sequences. A small one-step offset's change in the indexes of the sum of the triangle numbers changes the result from square to oblong and vice versa.

This was the reason Ramanujan arrived in a sum of positive Integer numbers with a nonsense fractional and negative result: $1 + 2 + 3 + 4 + \ldots = -\frac{1}{12}$.

6.5 Specific paraboctys notation

Because just 3 consecutive and constant central elements from column Zero $[Y[-1], Y[0], Y[1]] = [x_1, x_2, x_3] = [\mathrm{g}, \mathrm{h}, \mathrm{i}]$ determine all the specific paraboctys table, then just the 3 consecutive and constant central elements $[Y[-1], Y[0], Y[1]] = [x_1, x_2, x_3] = [\mathrm{g}, \mathrm{h}, \mathrm{i}]$ determine any specific paraboctys.

6.6 General paraboctys notation

See, the way we defined the specific paraboctys every vertical sequence in column $C+1$ is the elements in the vertical sequence of column C added by one unit.

To generalize any kind of paraboctys, let's allow the 3 lines Y[-1], Y[0], and Y[1] generating the quadratic equations to be any mathematical function where the index of the function is the value of the column. Thus, $Y[-1] = X_1(x)$, $Y[0] = X_2(x)$, and $Y[1] = X_3(x)$:

$$PS\big[Y[-1], Y[0], Y[1]\big] \text{ or } PS[X_1(x), X_2(x), X_3(x)]$$

Example 1: let's be

$$Y[1] = X_3(x) = x^2 - x + 11$$
$$Y[0] = X_2(x) = x^2 - x + 11$$
$$Y[-1] = X_1(x) = x^2 - x + 13$$

Then, we position each vertical at an x coordinate. Each vertical will have its elements applying the equation $Y[y] = \left(\frac{x_1 - 2x_2 + x_3}{2}\right) y^2 + \left(\frac{x_3 - x_1}{2}\right) y + x_2$ to its 3 consecutive elements in $Y[-1], Y[0], Y[1]$. See what we get:

	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x_ip	251	221	193	167	143	121	101	82,8	66,8	52,8	40,8	30,8	22,8	16,8	12,8	10,8	10,8	12,8	16,8	22,8	30,8	40,8	52,8	66,8	82,8	101	121	143	167	193	221
x_focus	251	221	193	167	143	121	101	83	67	53	41	31	23	17	13	11	11	13	17	23	31	41	53	67	83	101	121	143	167	193	221
LR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Δ	####	-883	-771	-667	-571	-483	-403	-331	-267	-211	-163	-123	-91	-67	-51	-43	-43	-51	-67	-91	-123	-163	-211	-267	-331	-403	-483	-571	-667	-771	-883
|√Δ|	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
C. G.	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root1	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root2	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root2-Root1	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Classif.	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES
y_ip	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
c	251	221	193	167	143	121	101	83	67	53	41	31	23	17	13	11	11	13	17	23	31	41	53	67	83	101	121	143	167	193	221
20	631	601	573	547	523	501	481	463	447	433	421	411	403	397	393	391	391	393	397	403	411	421	433	447	463	481	501	523	547	573	601
19	593	563	535	509	485	463	443	425	409	395	383	373	365	359	355	353	353	355	359	365	373	383	395	409	425	443	463	485	509	535	563
18	557	527	499	473	449	427	407	389	373	359	347	337	329	323	319	317	317	319	323	329	337	347	359	373	389	407	427	449	473	499	527
17	523	493	465	439	415	393	373	355	339	325	313	303	295	289	285	283	283	285	289	295	303	313	325	339	355	373	393	415	439	465	493
16	491	461	433	407	383	361	341	323	307	293	281	271	263	257	253	251	251	253	257	263	271	281	293	307	323	341	361	383	407	433	461
15	461	431	403	377	353	331	311	293	277	263	251	241	233	227	223	221	221	223	227	233	241	251	263	277	293	311	331	353	377	403	431
14	433	403	375	349	325	303	283	265	249	235	223	213	205	199	195	193	193	195	199	205	213	223	235	249	265	283	303	325	349	375	403
13	407	377	349	323	299	277	257	239	223	209	197	187	179	173	169	167	167	169	173	179	187	197	209	223	239	257	277	299	323	349	377
12	383	353	325	299	275	253	233	215	199	185	173	163	155	149	145	143	143	145	149	155	163	173	185	199	215	233	253	275	299	325	353
11	361	331	303	277	253	231	211	193	177	163	151	141	133	127	123	121	121	123	127	133	141	151	163	177	193	211	231	253	277	303	331
10	341	311	283	257	233	211	191	173	157	143	131	121	113	107	103	101	101	103	107	113	121	131	143	157	173	191	211	233	257	283	311
9	323	293	265	239	215	193	173	155	139	125	113	103	95	89	85	83	83	85	89	95	103	113	125	139	155	173	193	215	239	265	293
8	307	277	249	223	199	177	157	139	123	109	97	87	79	73	69	67	67	69	73	79	87	97	109	123	139	157	177	199	223	249	277
7	293	263	235	209	185	163	143	125	109	95	83	73	65	59	55	53	53	55	59	65	73	83	95	109	125	143	163	185	209	235	263
6	281	251	223	197	173	151	131	113	97	83	71	61	53	47	43	41	41	43	47	53	61	71	83	97	113	131	151	173	197	223	251
5	271	241	213	187	163	141	121	103	87	73	61	51	43	37	33	31	31	33	37	43	51	61	73	87	103	121	141	163	187	213	241
4	263	233	205	179	155	133	113	95	79	65	53	43	35	29	25	23	23	25	29	35	43	53	65	79	95	113	133	155	179	205	233
3	257	227	199	173	149	127	107	89	73	59	47	37	29	23	19	17	17	19	23	29	37	47	59	73	89	107	127	149	173	199	227
2	253	223	195	169	145	123	103	85	69	55	43	33	25	19	15	13	13	15	19	25	33	43	55	69	85	103	123	145	169	195	223
Y[1] 1	251	221	193	167	143	121	101	83	67	53	41	31	23	17	13	11	11	13	17	23	31	41	53	67	83	101	121	143	167	193	221
Y[0] 0	251	221	193	167	143	121	101	83	67	53	41	31	23	17	13	11	11	13	17	23	31	41	53	67	83	101	121	143	167	193	221
Y[-1] -1	253	223	195	169	145	123	103	85	69	55	43	33	25	19	15	13	13	15	19	25	33	43	55	69	85	103	123	145	169	195	223
-2	257	227	199	173	149	127	107	89	73	59	47	37	29	23	19	17	17	19	23	29	37	47	59	73	89	107	127	149	173	199	227
-3	263	233	205	179	155	133	113	95	79	65	53	43	35	29	25	23	23	25	29	35	43	53	65	79	95	113	133	155	179	205	233
-4	271	241	213	187	163	141	121	103	87	73	61	51	43	37	33	31	31	33	37	43	51	61	73	87	103	121	141	163	187	213	241
-5	281	251	223	197	173	151	131	113	97	83	71	61	53	47	43	41	41	43	47	53	61	71	83	97	113	131	151	173	197	223	251
-6	293	263	235	209	185	163	143	125	109	95	83	73	65	59	55	53	53	55	59	65	73	83	95	109	125	143	163	185	209	235	263
-7	307	277	249	223	199	177	157	139	123	109	97	87	79	73	69	67	67	69	73	79	87	97	109	123	139	157	177	199	223	249	277
-8	323	293	265	239	215	193	173	155	139	125	113	103	95	89	85	83	83	85	89	95	103	113	125	139	155	173	193	215	239	265	293
-9	341	311	283	257	233	211	191	173	157	143	131	121	113	107	103	101	101	103	107	113	121	131	143	157	173	191	211	233	257	283	311
-10	361	331	303	277	253	231	211	193	177	163	151	141	133	127	123	121	121	123	127	133	141	151	163	177	193	211	231	253	277	303	331
-11	383	353	325	299	275	253	233	215	199	185	173	163	155	149	145	143	143	145	149	155	163	173	185	199	215	233	253	275	299	325	353
-12	407	377	349	323	299	277	257	239	223	209	197	187	179	173	169	167	167	169	173	179	187	197	209	223	239	257	277	299	323	349	377
-13	433	403	375	349	325	303	283	265	249	235	223	213	205	199	195	193	193	195	199	205	213	223	235	249	265	283	303	325	349	375	403
-14	461	431	403	377	353	331	311	293	277	263	251	241	233	227	223	221	221	223	227	233	241	251	263	277	293	311	331	353	377	403	431
-15	491	461	433	407	383	361	341	323	307	293	281	271	263	257	253	251	251	253	257	263	271	281	293	307	323	341	361	383	407	433	461
-16	523	493	465	439	415	393	373	355	339	325	313	303	295	289	285	283	283	285	289	295	303	313	325	339	355	373	393	415	439	465	493
-17	557	527	499	473	449	427	407	389	373	359	347	337	329	323	319	317	317	319	323	329	337	347	359	373	389	407	427	449	473	499	527
-18	593	563	535	509	485	463	443	425	409	395	383	373	365	359	355	353	353	355	359	365	373	383	395	409	425	443	463	485	509	535	563
-19	631	601	573	547	523	501	481	463	447	433	421	411	403	397	393	391	391	393	397	403	411	421	433	447	463	481	501	523	547	573	601
-20	671	641	613	587	563	541	521	503	487	473	461	451	443	437	433	431	431	433	437	443	451	461	473	487	503	521	541	563	587	613	641

Figure 1. The paraboctys $PS[x^2 - x + 13, x^2 - x + 11, x^2 - x + 11]$.

Example 2: let's be

$$Y[1] = X_3(x) = x$$
$$Y[0] = X_2(x) = 0$$
$$Y[-1] = X_1(x) = -x$$

		-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x_ip		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
x_focus		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
LR		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Δ		225	196	169	144	121	100	81	64	49	36	25	16	9	4	1	0	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
|√Δ|		15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C. G.		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Root1		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root2		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root2-Root1		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Classif.		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
y_ip		#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####	#####
f		####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
a		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b		-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	-300	-280	-260	-240	-220	-200	-180	-160	-140	-120	-100	-80	-60	-40	-20	0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300
	19	-285	-266	-247	-228	-209	-190	-171	-152	-133	-114	-95	-76	-57	-38	-19	0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285
	18	-270	-252	-234	-216	-198	-180	-162	-144	-126	-108	-90	-72	-54	-36	-18	0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270
	17	-255	-238	-221	-204	-187	-170	-153	-136	-119	-102	-85	-68	-51	-34	-17	0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255
	16	-240	-224	-208	-192	-176	-160	-144	-128	-112	-96	-80	-64	-48	-32	-16	0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240
	15	-225	-210	-195	-180	-165	-150	-135	-120	-105	-90	-75	-60	-45	-30	-15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225
	14	-210	-196	-182	-168	-154	-140	-126	-112	-98	-84	-70	-56	-42	-28	-14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
	13	-195	-182	-169	-156	-143	-130	-117	-104	-91	-78	-65	-52	-39	-26	-13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
	12	-180	-168	-156	-144	-132	-120	-108	-96	-84	-72	-60	-48	-36	-24	-12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
	11	-165	-154	-143	-132	-121	-110	-99	-88	-77	-66	-55	-44	-33	-22	-11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
	10	-150	-140	-130	-120	-110	-100	-90	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
	9	-135	-126	-117	-108	-99	-90	-81	-72	-63	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
	8	-120	-112	-104	-96	-88	-80	-72	-64	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
	7	-105	-98	-91	-84	-77	-70	-63	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
	6	-90	-84	-78	-72	-66	-60	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
	5	-75	-70	-65	-60	-55	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
	4	-60	-56	-52	-48	-44	-40	-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
	3	-45	-42	-39	-36	-33	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
	2	-30	-28	-26	-24	-22	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
Y[1]	1	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y[0]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y[-1]	-1	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
	-2	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20	-22	-24	-26	-28	-30
	-3	45	42	39	36	33	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	-30	-33	-36	-39	-42	-45
	-4	60	56	52	48	44	40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36	-40	-44	-48	-52	-56	-60
	-5	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75
	-6	90	84	78	72	66	60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54	-60	-66	-72	-78	-84	-90
	-7	105	98	91	84	77	70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56	-63	-70	-77	-84	-91	-98	-105
	-8	120	112	104	96	88	80	72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56	-64	-72	-80	-88	-96	-104	-112	-120
	-9	135	126	117	108	99	90	81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63	-72	-81	-90	-99	-108	-117	-126	-135
	-10	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	0	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100	-110	-120	-130	-140	-150
	-11	165	154	143	132	121	110	99	88	77	66	55	44	33	22	11	0	-11	-22	-33	-44	-55	-66	-77	-88	-99	-110	-121	-132	-143	-154	-165
	-12	180	168	156	144	132	120	108	96	84	72	60	48	36	24	12	0	-12	-24	-36	-48	-60	-72	-84	-96	-108	-120	-132	-144	-156	-168	-180
	-13	195	182	169	156	143	130	117	104	91	78	65	52	39	26	13	0	-13	-26	-39	-52	-65	-78	-91	-104	-117	-130	-143	-156	-169	-182	-195
	-14	210	196	182	168	154	140	126	112	98	84	70	56	42	28	14	0	-14	-28	-42	-56	-70	-84	-98	-112	-126	-140	-154	-168	-182	-196	-210
	-15	225	210	195	180	165	150	135	120	105	90	75	60	45	30	15	0	-15	-30	-45	-60	-75	-90	-105	-120	-135	-150	-165	-180	-195	-210	-225
	-16	240	224	208	192	176	160	144	128	112	96	80	64	48	32	16	0	-16	-32	-48	-64	-80	-96	-112	-128	-144	-160	-176	-192	-208	-224	-240
	-17	255	238	221	204	187	170	153	136	119	102	85	68	51	34	17	0	-17	-34	-51	-68	-85	-102	-119	-136	-153	-170	-187	-204	-221	-238	-255
	-18	270	252	234	216	198	180	162	144	126	108	90	72	54	36	18	0	-18	-36	-54	-72	-90	-108	-126	-144	-162	-180	-198	-216	-234	-252	-270
	-19	285	266	247	228	209	190	171	152	133	114	95	76	57	38	19	0	-19	-38	-57	-76	-95	-114	-133	-152	-171	-190	-209	-228	-247	-266	-285
	-20	300	280	260	240	220	200	180	160	140	120	100	80	60	40	20	0	-20	-40	-60	-80	-100	-120	-140	-160	-180	-200	-220	-240	-260	-280	-300

Figure 1. The paraboctys $PS[-x, 0, x]$. It is the hyperboctys $HS[0,0,0]$, which is the FMT Full Multiplication Table.

Example 3: let's be

$$Y[1] = X_3(x) = x$$
$$Y[0] = X_2(x) = x$$
$$Y[-1] = X_1(x) = x + 2$$

	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x_ip	-15	-14	-13	-12	-11	-10	-9,3	-8,3	-7,3	-6,3	-5,3	-4,3	-3,3	-2,3	-1,3	-0,3	0,75	1,75	2,75	3,75	4,75	5,75	6,75	7,75	8,75	9,75	10,8	11,8	12,8	13,8	14,8
x_focus	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
LR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Δ	61	57	53	49	45	41	37	33	29	25	21	17	13	9	5	1	-3	-7	-11	-15	-19	-23	-27	-31	-35	-39	-43	-47	-51	-55	-59
|√Δ|	7,81	7,55	7,28	7	6,71	6,4	6,08	5,74	5,39	5	4,58	4,12	3,61	3	2,24	1	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
C. G.	0,81	0,55	0,28	0	0,71	0,4	0,08	0,74	0,39	0	0,58	0,12	0,61	0	0,24	0	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root1	4,41	4,27	4,14	4	3,85	3,7	3,54	3,37	3,19	3	2,79	2,56	2,3	2	1,62	1	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root2	-3,4	-3,3	-3,1	-3	-2,9	-2,7	-2,5	-2,4	-2,2	-2	-1,8	-1,6	-1,3	-1	-0,6	0	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Root2-Root1	-7,8	-7,5	-7,3	-7	-6,7	-6,4	-6,1	-5,7	-5,4	-5	-4,6	-4,1	-3,6	-3	-2,2	-1	####	####	####	####	####	####	####	####	####	####	####	####	####	####	####
Classif.	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES	DES
y_ip	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
b	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
c	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
20	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395
19	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357
18	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321
17	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287
16	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255
15	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225
14	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197
13	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171
12	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147
11	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125
10	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
9	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87
8	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
7	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
6	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
5	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
3	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Y[1] 1	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y[0] 0	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y[-1] -1	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
-2	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
-3	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
-4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
-5	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
-6	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
-7	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
-8	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87
-9	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
-10	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125
-11	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147
-12	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171
-13	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197
-14	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225
-15	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255
-16	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287
-17	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321
-18	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357
-19	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395
-20	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435

Figure 1. The paraboctys $PS[x+2, x, x]$. It is the specific paraboctys $PS[2,0,0]$, which is the paraboctys of the initial reasoning of our studies.

Example 4: let's be

$$Y[1] = X_3(x) = x$$
$$Y[0] = X_2(x) = x$$
$$Y[-1] = X_1(x) = x$$

a		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
c		-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	20	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	19	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	18	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	17	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	15	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	14	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	13	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	11	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	10	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	9	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	8	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	7	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	6	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	5	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	4	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	3	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	2	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y[1]	1	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y[0]	0	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Y[-1]	-1	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-2	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-3	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-4	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-5	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-6	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-7	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-8	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-9	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-10	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-11	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-12	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-13	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-14	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-15	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-17	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-18	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-19	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	-20	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Figure 1. The paraboctys $PS[x, x, x]$.

Conclusion:

To be complete and flexible, all paraboctys will have their notation as 3 consecutive functions. Each set of 3 consecutive functions will vary according to the column of the paraboctys. They will always be an X-axis function.

For any 3 functions where $Y[-1] = X_1(x), Y[0] = X_2(x), Y[1] = X_3(x)$, then the generic and final paraboctys notation will be:

$$PS\big[Y[-1], Y[0], Y[1]\big] = PS[X_1(x), X_2(x), X_3(x)]$$

7 Quadratics on all diagonals and verticals of specific paraboctys

We can do the same reasoning that we did for verticals for any diagonal of any inclination. The method of finite differences guarantees the same polynomial degree.

The reason is that the horizontal rows are the sequences of consecutive Integer numbers. Thus, whatever element we choose as the central point to rotate the diagonal, we will subtract the same amount that we add above the central point from the same value of the equidistant elements below the central point.

See the mechanism in the figures below for specific paraboctys.

The change of the coefficient a will only change if there are index jumps in the diagonals. Then, the coefficient will vary by following the square of the index jump.

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		a	a	a	a	a	a	a	a	a
b		-a	-a	-a	-a	-a	-a	-a	-a	-a
c		-4	-3	-2	-1	0	1	2	3	4
	4	12a-4	12a-3	12a-2	12a-1	12a	12a+1	12a+2	12a+3	12a+4
	3	6a-4	6a-3	6a-2	6a-1	6a	6a+1	6a+2	6a+3	6a+4
	2	2a-4	2a-3	2a-2	2a-1	2a	2a+1	2a+2	2a+3	2a+4
Y[1]=x_3	1	4	3	2	1	0	1	2	3	4
Y[0]=x_2	0	4	3	2	1	0	1	2	3	4
Y[-1]=x_1	-1	2a-4	2a-3	2a-2	2a-1	2a	2a+1	2a+2	2a+3	2a+4
	-2	6a-4	6a-3	6a-2	6a-1	6a	6a+1	6a+2	6a+3	6a+4
	-3	12a-4	12a-3	12a-2	12a-1	12a	12a+1	12a+2	12a+3	12a+4
	-4	20a-4	20a-3	20a-2	20a-1	20a	20a+1	20a+2	20a+3	20a+4

Figure 1. $PS[x + 2a, x, x]$. All diagonals with the coefficient of second-degree terms a.

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		a	a	a	a	a	a	a	a	a
b		-a	-a	-a	-a	-a	-a	-a	-a	-a
c		-4	-3	-2	-1	0	1	2	3	4
	4	12a-4	12a-3	12a-2	12a-1	12a	12a+1	12a+2	12a+3	12a+4
	3	6a-4	6a-3	6a-2	6a-1	6a	6a+1	6a+2	6a+3	6a+4
	2	2a-4	2a-3	2a-2	2a-1	2a	2a+1	2a+2	2a+3	2a+4
Y[1]=x_3	1	4	3	2	1	0	1	2	3	4
Y[0]=x_2	0	4	3	2	1	0	1	2	3	4
Y[-1]=x_1	-1	2a-4	2a-3	2a-2	2a-1	2a	2a+1	2a+2	2a+3	2a+4
	-2	6a-4	6a-3	6a-2	6a-1	6a	6a+1	6a+2	6a+3	6a+4
	-3	12a-4	12a-3	12a-2	12a-1	12a	12a+1	12a+2	12a+3	12a+4
	-4	20a-4	20a-3	20a-2	20a-1	20a	20a+1	20a+2	20a+3	20a+4

Figure 1. $PS[x + 2a, x, x]$. All diagonals with the coefficient of second-degree terms $4a$.

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		a	a	a	a	a	a	a	a	a
b		−a	−a	−a	−a	−a	−a	−a	−a	−a
c		-4	-3	-2	-1	0	1	2	3	4
	4	12a−4	12a−3	12a−2	12a−1	12a	12a+1	12a+2	12a+3	12a+4
	3	6a−4	6a−3	6a−2	6a−1	6a	6a+1	6a+2	6a+3	6a+4
	2	2a−4	2a−3	2a−2	2a−1	2a	2a+1	2a+2	2a+3	2a+4
Y[1]=x_3	1	4	3	2	1	0	1	2	3	4
Y[0]=x_2	0	4	3	2	1	0	1	2	3	4
Y[-1]=x_1	-1	2a−4	2a−3	2a−2	2a−1	2a	2a+1	2a+2	2a+3	2a+4
	-2	6a−4	6a−3	6a−2	6a−1	6a	6a+1	6a+2	6a+3	6a+4
	-3	12a−4	12a−3	12a−2	12a−1	12a	12a+1	12a+2	12a+3	12a+4
	-4	20a−4	20a−3	20a−2	20a−1	20a	20a+1	20a+2	20a+3	20a+4

Figure 1. $PS[x+2a,x,x]$. All diagonals with the coefficient of second-degree terms $9a$.

Columns C --->		-4	-3	-2	-1	0	1	2	3	4
a		a	a	a	a	a	a	a	a	a
b		−a	−a	−a	−a	−a	−a	−a	−a	−a
c		-4	-3	-2	-1	0	1	2	3	4
	4	12a−4	12a−3	12a−2	12a−1	12a	12a+1	12a+2	12a+3	12a+4
	3	6a−4	6a−3	6a−2	6a−1	6a	6a+1	6a+2	6a+3	6a+4
	2	2a−4	2a−3	2a−2	2a−1	2a	2a+1	2a+2	2a+3	2a+4
Y[1]=x_3	1	4	3	2	1	0	1	2	3	4
Y[0]=x_2	0	4	3	2	1	0	1	2	3	4
Y[-1]=x_1	-1	2a−4	2a−3	2a−2	2a−1	2a	2a+1	2a+2	2a+3	2a+4
	-2	6a−4	6a−3	6a−2	6a−1	6a	6a+1	6a+2	6a+3	6a+4
	-3	12a−4	12a−3	12a−2	12a−1	12a	12a+1	12a+2	12a+3	12a+4
	-4	20a−4	20a−3	20a−2	20a−1	20a	20a+1	20a+2	20a+3	20a+4

Figure 1. $PS[x+2a,x,x]$. All diagonals with the coefficient of second-degree terms $16a$.

8 Definition of "trianz"

Rightly, many Editors dislike the use of the term "Tetractys" different from the real original definition: "*The Tetractys (Greek: τετρακτύς), or tetrad, or the Tetractys of the decade is a triangular figure consisting of ten points arranged in four rows: one, two, three, and four points in each row, which is the geometrical representation of the fourth triangular number*".

Although it was a source of inspiration for the name paraboctys, the triangles that appear in our specific paraboctys table have differences from the original Tetractys.

First, our triangular figure has an infinite amount of elements arranged in a triangular grid, in the same way as Klauber's triangle. The Tetractys have a finite amount of elements.

We show Tetractys only in the form of an isometric lattice grid. In our studies, we should study the triangles of the paraboctys both in the isometric lattice grid and in the Cartesian lattice grid. When in the isometric lattice grid, we should consider each cell being at each vertex of the equilateral triangles and/or within the triangles.

Also, because of the Offset theory, in our infinite triangles, we need to change the initial value and its position. Tetractys and Klauber's triangle start rigidly with number One at the top.

In our study, we need to have the flexibility to start our triangle in any row and with any Integer, including the Positive, Zero, and the Negative.

Note that, Klauber's triangle and Tetractys have the characteristic of increasing 2 elements at each line. But both start from just the Unit element. This forces us to have in all rows only an Odd number of elements. In this study, we will open the possibility of creating triangles with an Even number of elements in each line, without changing the increment rate of 2 elements to each line.

Only with the flexibility of values and flexibility of origin locations we can cover all the possibilities of finding Prime number sequences.

Later we see that we can base some triangle formats on the repunit's powers format.

In this line of reasoning, we will define a triangle called "trianz" as having the format of the specific paraboctys' triangles, but starting at Zero instead of One. All trianz have all integers once.

Trianz name with z at the end to be Zero mnemonic.

The primary reason to start with the number Zero is because of the Composite Generators that sift all the sequences of the prime numbers in the paraboctys.

8.1 The trianz notation

Because these triangles have their properties depending on the paraboctys, then we will use a notation similar to the paraboctys.

For any 3 consecutive functions where $Y[-1] = X_1(x), Y[0] = X_2(x), Y[1] = X_3(x)$, then the generic and final trianz notation will be:

$$TZ[Y[-1], Y[0], Y[1]] = TZ[X_1(x), X_2(x), X_3(x)]$$

8.2 Specific trianz's in specific paraboctys where coefficient $a = 1$

For coefficient $a = 1$, there are only two types of specific paraboctys: $PS[x+1, x, x+1]$ and $PS[x+2, x, x]$. Consequently, there are only two types of specific trianz's $TZ[x+1, x, x+1]$ and $TZ[x+2, x, x]$ respectively. All other specific paraboctys and specific trianz with quadratics for $a = 1$ will be an offset of them.

See the distribution of the countless trianz $TZ[x+1, x, x+1]$ in paraboctys $PS[x+1, x, x+1]$.

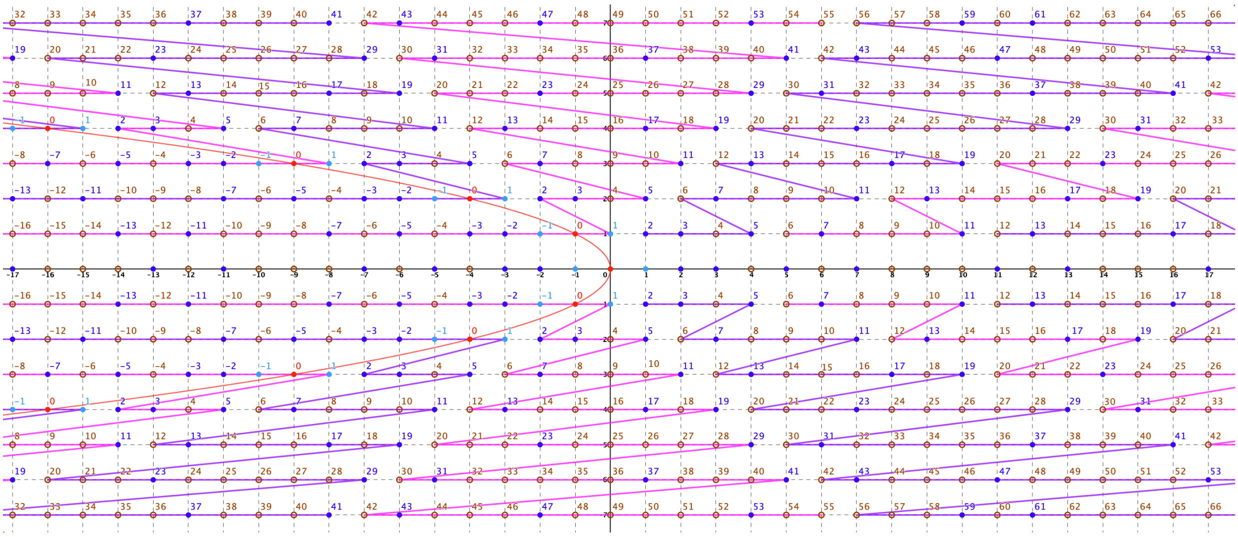


Figure 1. Trianz $TZ[x+1, x, x+1]$ in paraboctys $PS[x+1, x, x+1]$.

See the distribution of the countless trianz $TZ[x+2, x, x]$ in paraboctys $PS[x+2, x, x]$.

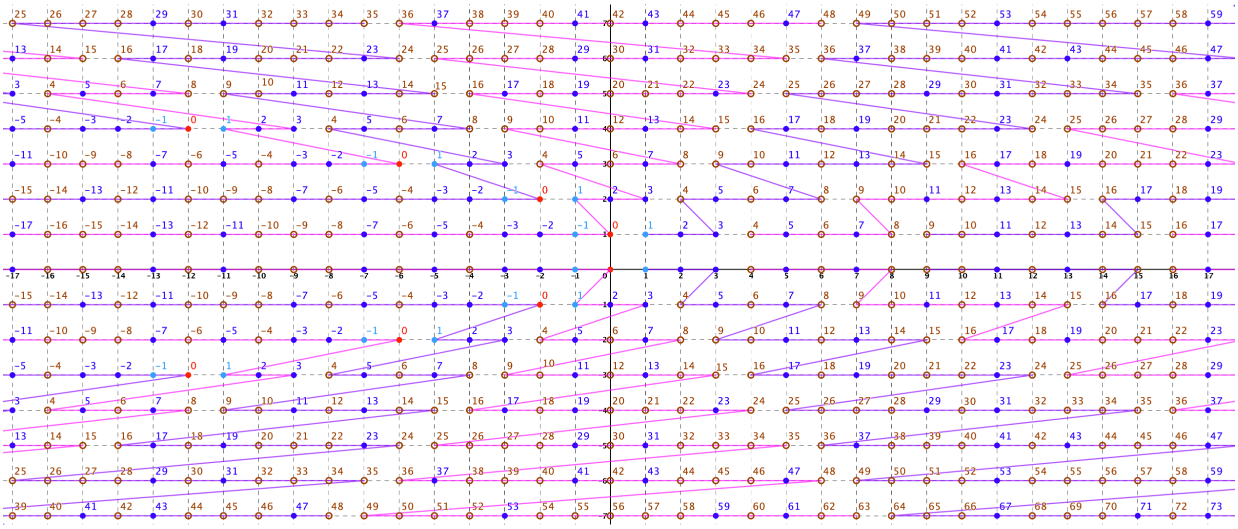


Figure 1. Trianz $TZ[x+2, x, x]$ in paraboctys $PS[x+2, x, x]$.

Note that to obey the rule of all trianz having all the Integer numbers once, then all the negative numbers and each Zero take part in two trianz's. See the example below:

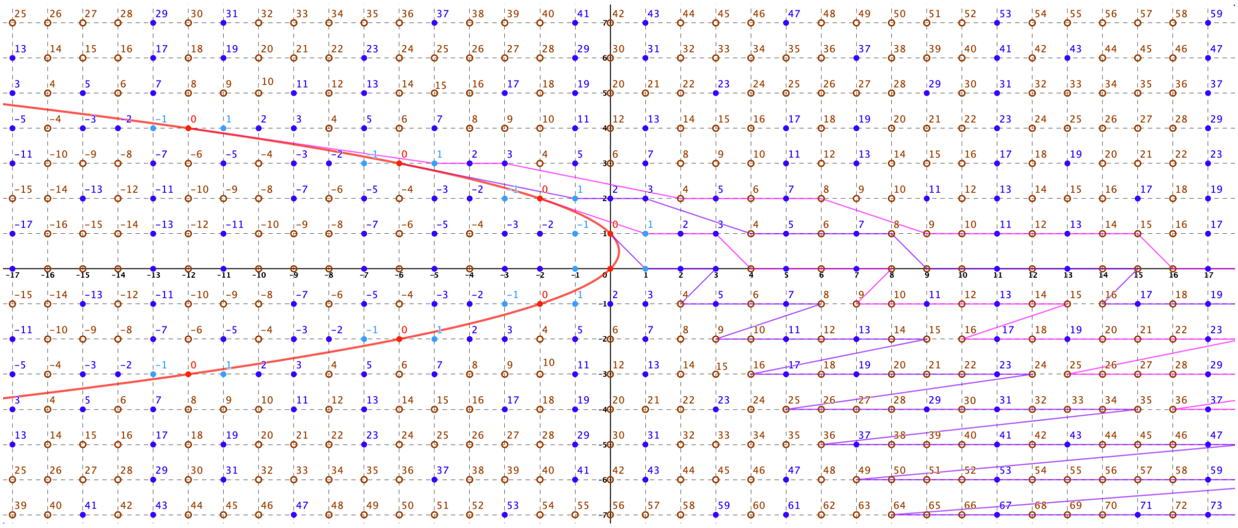


Figure 1. Each Zero participates in two trianz's: one up and one down.

8.3 Trianz offset

Let's define the offset value of each trianz as the value of the row where the element Zero of the central element line of the trianz is located.

This way, the down-facing trianz on the right side of the paraboctys center line will have a positive offset. The down-facing trianz on the left side of the paraboctys center line will have a negative offset. The opposite occurs for the up-facing trianz.

See the central trianz's in table format in paraboctys $PS[x+1,x,x+1]$ and $PS[x+2,x,x]$:

11	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131
10		90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	
9			72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89		
8				56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71			
7					42	43	44	45	46	47	48	49	50	51	52	53	54	55				
6						30	31	32	33	34	35	36	37	38	39	40	41					
5							20	21	22	23	24	25	26	27	28	29						
4								12	13	14	15	16	17	18	19							
3									6	7	8	9	10	11								
2										2	3	4	5									
1	-10	-9	-8	7	-6	-5	-4	-3	-2	-1	0	1										
0												0										
-1	-10	-9	-8	7	-6	-5	-4	-3	-2	-1	0	1										
-2										2	3	4	5									
-3									6	7	8	9	10	11								
-4								12	13	14	15	16	17	18	19							
-5							20	21	22	23	24	25	26	27	28	29						
-6						30	31	32	33	34	35	36	37	38	39	40	41					
-7					42	43	44	45	46	47	48	49	50	51	52	53	54	55				
-8				56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71			
-9			72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89		
-10		90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	

11	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
10		81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	
9			64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80		
8				49	50	51	52	53	54	55	56	57	58	59	60	61	62	63			
7					36	37	38	39	40	41	42	43	44	45	46	47	48				
6						25	26	27	28	29	30	31	32	33	34	35					
5							16	17	18	19	20	21	22	23	24						
4								9	10	11	12	13	14	15							
3									4	5	6	7	8								
2										1	2	3									
1	-10	-9	-8	7	-6	-5	-4	-3	-2	-1	0										
0	-10	-9	-8	7	-6	-5	-4	-3	-2	-1	0										
-1										1	2	3									
-2									4	5	6	7	8								
-3								9	10	11	12	13	14	15							
-4							16	17	18	19	20	21	22	23	24						
-5						25	26	27	28	29	30	31	32	33	34	35					
-6					36	37	38	39	40	41	42	43	44	45	46	47	48				
-7				49	50	51	52	53	54	55	56	57	58	59	60	61	62	63			
-8			64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80		
-9		81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	
-10	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120

Figure 1. The central trianz $TZ[x+1,x,x+1]$ and trianz $TZ[x+2,x,x]$ in table format.

In Figure 1. above, the trianz $TZ[x+1,x,x+1]$ facing down has offset $f=-1$. The trianz $TZ[x+1,x,x+1]$ facing up has offset $f=1$. The trianz $TZ[x+2,x,x]$ face down has offset $f=0$. The trianz $TZ[x+2,x,x]$ face up has offset $f=1$.

Thus, the trianz $TZ[x+1,x,x+1]$ offset $f=0$ is a straight line representing the line of integers.

8.4 Reading the trianz

The advantage of defining the trianz object appears when we study $TZ[x+2,x,x]$.

There are infinitely many trianz $TZ[x+2,x,x]$ in offset in paraboctys $PS[x+2,x,x]$. Let's focus on the center where the offset is zero.

The Oblong sequence divides $TZ[x+2,x,x]$ into positive columns on the right side and negative columns on the left side.

Any diagonal or vertical line in paraboctys will cross the rows $\left[Y[-1],Y[0],Y[1]\right]$. From the value of these 3 elements given by $\left[Y[-1],Y[0],Y[1]\right]$ we can use our general quadratic formula to get the corresponding quadratic equation of the respective line.

For example, in the bottom trianz $TZ[x+2,x,x]$:

1. It has left an ascendant diagonal line $x=y$. This diagonal line passes through $[Y[-1]=1,Y[0]=0,Y[1]=1]$. So, this diagonal has the quadratic elements $[1,0,1] \equiv Y[y]=y^2$, A000290 Square numbers in offset $f=0$.
2. It has the right descendant diagonal line $x=-y$. This diagonal line passes through $[Y[-1]=3,Y[0]=0,Y[1]=-1]$. So, this diagonal has quadratic elements $[3,0,-1] \equiv Y[y]=y^2-2y$, A005563 (Square minus One) numbers in offset $f=1$.
3. Both the bottom and the top trianz have in the center the same vertical line in column $x=0$. This vertical line passes through $[Y[-1]=2,Y[0]=0,Y[1]=0]$. So, this diagonal has the quadratic elements $[2,0,0] \equiv X[y]=y^2-y$, A002378 Oblong numbers in offset $f=0$.

The top trianz $TZ[x+2,x,x]$ has equivalent features, but an offset one from the other.

4. The descendant diagonal $-1:1 \equiv -45^{\circ}$ in the top's left trianz $TZ[x+2,x,x]$, is the line with equation $x=-y+1$. This line represents the quadratics given by $[4,1,0] \equiv X[y]=y^2-2y+1$, A000290 Square numbers in offset $f=1$.
5. The ascendant diagonal $1:1 \equiv 45^{\circ}$ in the top's right trianz $TZ[x+2,x,x]$, is the line with equation $x=y-1$. This line represents the quadratics given by $[0,-1,0] \equiv X[y]=y^2-1$, A005563 (Square minus One) numbers in offset $f=0$.

9 Primes in the trianz

See that the vertical line passing through the Zero of $TZ[x+2,x,x]$ is the vertical line at column $x=0$ in $PS[x+2,x,x]$. This line represents the quadratics $[2,0,0] \equiv X[y]=y^2-y$, which is the A002378 Oblong quadratic.

So, each row of $TZ[x+2,x,x]$ starts in A000290 Square numbers, passes through A002378 Oblong numbers in the center, and ends at A005563 (Square Minus One) numbers.

$TZ[x+2,x,x]$ shows us that between two consecutive Oblong numbers, there is one Square number. And, between two consecutive Square numbers, there is one Oblong number.

Having equidistant elements with Oblong numbers in the middle, allow us to write:

$$Oblong[y]=\frac{Square[y-1]+(Square[y]-1)}{2}=\frac{(y-1)^2+(y^2-1)}{2}=y^2-y$$

See the extended study at chapter triangles in paraboctys (from the former draft: https://oeis.org/wiki/User:Charles_Kusniec/Triangles_in_Paraboctys).

So, the vertical A002378 Oblong numbers in the central column of $PS[x+2,x,x]$ divide $TZ[x+2,x,x]$ into two halves. The trianz $TZ[x+1,x,x+1]$ has an equivalent division. See below:

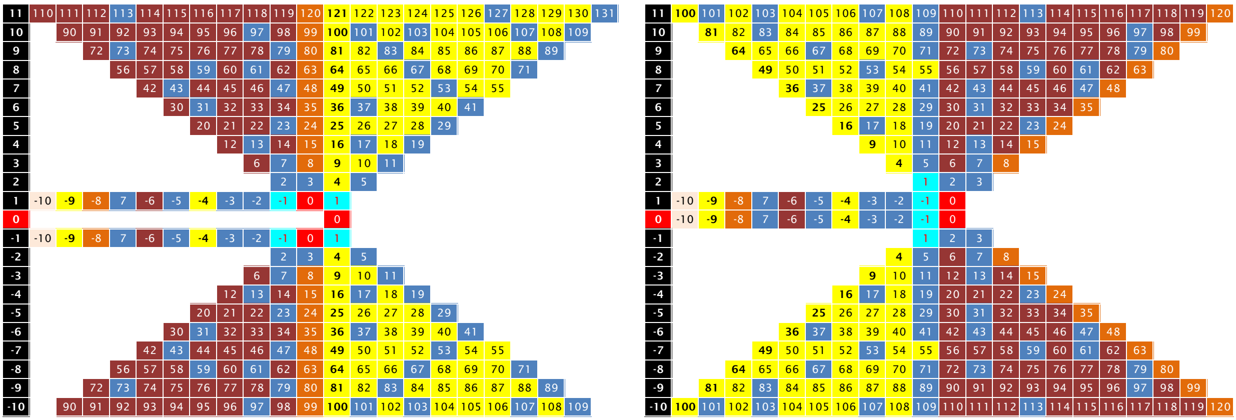


Figure 1. The trianz $TZ[x+1,x,x+1]$ and $TZ[x+2,x,x]$ divided into two areas: the area of the squares in yellow and the area of the oblong ones in maroon.

9.1 Primes between a Square number and its following Oblong number

Primes in the yellow area of $TZ[x+1, x, x+1]$ and $TZ[x+2, x, x]$ are Primes between a Square and its following Oblong. They seem to be the sequence A307508 Primes p for which the continued fraction expansion of sqrt(p) does not have a 1 in the second position. {2, 5, 11, 17, 19, 29, 37, 41, 53, 67, 71, 83, 89, 101, 103, 107, 109, 127, 131, 149, 151, 173, 179, 181, 197, 199, 227, 229, 233, 239, 257, 263, 269, 271, 293, 331, 337, 367, 373, 379, 401, 409, 419, 443, 449, 457, 461, 487, 491, 499, 503, 541, 547, 577, 587, 593, 599, …}.

Here the doubt falls on prime 2. In $TZ[x+1, x, x+1]$ prime 2 seems to be clear in the maroon Oblong area, while in $TZ[x+2, x, x]$ it appears exactly on the dividing line between the Square and Oblong areas.

However, if we consider the number of Primes between a Square number and its following Oblong number given by the sequence A089610 Number of Primes between n^2 and $\left(n+\frac{1}{2}\right)^2$, then prime 2 belongs to the yellow area of the Squares.

The sequence A089610 Number of Primes between n^2 and $\left(n+\frac{1}{2}\right)^2$ is {1, 1, 1, 2, 1, 2, 1, 2, 2, 4, 2, 2, 3, 2, 4, 4, 1, 2, 3, 3, 4, 4, 2, 4, 4, 4, 4, 4, 4, 4, 5, 5, 6, 4, 5, 7, 3, 6, 6, 8, 5, 5, 7, 4, 6, 7, 6, 7, 6, 6, 5, 9, 7, 7, 6, 7, 7, 6, 8, 8, 7, 7, 8, 9, 11, 7, 8, 10, 8, 11, 8, 7, 7, 10, 11, 12, 4, 9, 11, 6, 9, 9, 10, 8, 9, 8, 11, 8, 8, 9, 10, 8, 13, 10, 9, 10, 14, 12,…}.

For small values of n, these numbers exhibit higher and lower values as n increases. Conjectures: After $n = 17$, $a(n) > 1$. There is a n_1 such that $a(n) < a(n+1)$ for all $n \geq n_1$. Same as the number of Primes between n^2 and $n^2 + n$. Oppermann conjectured in 1882 that $a(n) > 0$.

9.2 Primes between an Oblong number and its following Square number

Primes in the maroon area of $TZ[x+1, x, x+1]$ and $TZ[x+2, x, x]$ are Primes between an Oblong and its following Square. They seem to be the sequence A334163 Primes p whose continued fraction expansion of $\sqrt{p}$ has a 1 in the second position. {3, 7, 13, 23, 31, 43, 47, 59, 61, 73, 79, 97, 113, 137, 139, 157, 163, 167, 191, 193, 211, 223, 241, 251, 277, 281, 283, 307, 311, 313, 317, 347, 349, 353, 359, 383, 389, 397, 421, 431, 433, 439, 463, 467, 479, 509, 521, 523, 557, 563, 569, 571, 601, 607, 613, 617, 619, …}.

Here the doubt falls on prime 3. In $TZ[x+1, x, x+1]$ prime 3 seems to be clear out of the maroon Oblong area, while in $TZ[x+2, x, x]$ it seems to be clear in the maroon Oblong area.

However, if we consider the number of Primes between an Oblong number and its following Square number given by the sequence A094189 Number of Primes between $n^2 - n$ and n^2 (inclusive), then prime 3 belongs to the maroon area of the Oblong.

The sequence A094189 Number of Primes between $n^2 - n$ and n^2 (inclusive) is {0, 2, 1, 1, 1, 1, 2, 2, 2, 1, 1, 2, 3, 2, 2, 2, 3, 4, 4, 3, 4, 3, 3, 4, 5, 4, 3, 4, 5, 4, 4, 5, 4, 4, 5, 5, 2, 6, 6, 5, 4, 6, 4, 5, 7, 7, 3, 7, 8, 4, 5, 10, 7, 5, 6, 5, 5, 10, 7, 8, 8, 6, 10, 7, 5, 5, 8, 7, 7, 5, 10, 7, 8, 10, 7, 7, 10, 10, 9, 12, 7, 11, 10, 10, 9, 7, 13, 11, 10, 10, 11, 10, 11, 10, 11,…}.

Conjecture: for $n > 11$, $a(n) > 1$.

9.3 Distribution of prime numbers between Square and Oblong and vice versa.

Conjecture: 50% of the Primes are between Square and Oblong and 50% between Oblong and Square.

row of TZ[2,0,0]	StO A089610 Number of primes between Square and Oblong	Sum StO Axxxxxx	OtS A094189 Number of primes between Oblong and Square	Sum OtS Axxxxxx	StO + OtS A014085 Number of primes between consecutive Squares	Sum StO + Sum OtS Axxxxxx All Primes until row	Sum StO / (Sum StO + Sum OtS)	Sum OtS / (Sum StO + Sum OtS)
2	1	1	1	1	2	2	50,000%	50,000%
3	1	2	1	2	2	4	50,000%	50,000%
4	1	3	1	3	2	6	50,000%	50,000%
5	2	5	1	4	3	9	55,556%	44,444%
6	1	6	1	5	2	11	54,545%	45,455%
7	2	8	2	7	4	15	53,333%	46,667%
8	1	9	2	9	3	18	50,000%	50,000%
9	2	11	2	11	4	22	50,000%	50,000%
10	2	13	1	12	3	25	52,000%	48,000%
11	4	17	1	13	5	30	56,667%	43,333%
12	2	19	2	15	4	34	55,882%	44,118%
13	2	21	3	18	5	39	53,846%	46,154%
14	3	24	2	20	5	44	54,545%	45,455%
15	2	26	2	22	4	48	54,167%	45,833%
16	4	30	2	24	6	54	55,556%	44,444%
17	4	34	3	27	7	61	55,738%	44,262%
18	1	35	4	31	5	66	53,030%	46,970%
19	2	37	4	35	6	72	51,389%	48,611%
20	3	40	3	38	6	78	51,282%	48,718%
21	3	43	4	42	7	85	50,588%	49,412%
22	4	47	3	45	7	92	51,087%	48,913%
23	4	51	3	48	7	99	51,515%	48,485%
24	2	53	4	52	6	105	50,476%	49,524%
25	4	57	5	57	9	114	50,000%	50,000%
26	4	61	4	61	8	122	50,000%	50,000%
27	4	65	3	64	7	129	50,388%	49,612%
28	4	69	4	68	8	137	50,365%	49,635%
29	4	73	5	73	9	146	50,000%	50,000%
30	4	77	4	77	8	154	50,000%	50,000%
31	4	81	4	81	8	162	50,000%	50,000%
32	5	86	5	86	10	172	50,000%	50,000%
33	5	91	4	90	9	181	50,276%	49,724%
34	6	97	4	94	10	191	50,785%	49,215%
35	4	101	5	99	9	200	50,500%	49,500%
36	5	106	5	104	10	210	50,476%	49,524%
37	7	113	2	106	9	219	51,598%	48,402%
38	3	116	6	112	9	228	50,877%	49,123%
39	6	122	6	118	12	240	50,833%	49,167%
40	6	128	5	123	11	251	50,996%	49,004%
41	8	136	4	127	12	263	51,711%	48,289%
42	5	141	6	133	11	274	51,460%	48,540%
43	5	146	4	137	9	283	51,590%	48,410%
44	7	153	5	142	12	295	51,864%	48,136%
45	4	157	7	149	11	306	51,307%	48,693%
46	6	163	7	156	13	319	51,097%	48,903%
47	7	170	3	159	10	329	51,672%	48,328%
48	6	176	7	166	13	342	51,462%	48,538%
49	7	183	8	174	15	357	51,261%	48,739%
50	6	189	4	178	10	367	51,499%	48,501%
51	6	195	5	183	11	378	51,587%	48,413%
52	5	200	10	193	15	393	50,891%	49,109%
53	9	209	7	200	16	409	51,100%	48,900%
54	7	216	5	205	12	421	51,306%	48,694%
55	7	223	6	211	13	434	51,382%	48,618%
56	6	229	5	216	11	445	51,461%	48,539%
57	7	236	5	221	12	457	51,641%	48,359%
58	7	243	10	231	17	474	51,266%	48,734%
59	6	249	7	238	13	487	51,129%	48,871%
60	8	257	8	246	16	503	51,093%	48,907%

Figure 1. StO means Square to Oblong, OtS means Oblong to Square. Comparison between rows from 2 to 60 of $TZ[x+2,x,x]$.

9942	553	2850420	565	2848314	1118	5698734	50,018%	49,982%
9943	523	2850943	522	2848836	1045	5699779	50,018%	49,982%
9944	550	2851493	532	2849368	1082	5700861	50,019%	49,981%
9945	550	2852043	518	2849886	1068	5701929	50,019%	49,981%
9946	526	2852569	534	2850420	1060	5702989	50,019%	49,981%
9947	535	2853104	531	2850951	1066	5704055	50,019%	49,981%
9948	526	2853630	520	2851471	1046	5705101	50,019%	49,981%
9949	522	2854152	546	2852017	1068	5706169	50,019%	49,981%
9950	547	2854699	548	2852565	1095	5707264	50,019%	49,981%
9951	553	2855252	564	2853129	1117	5708381	50,019%	49,981%
9952	541	2855793	576	2853705	1117	5709498	50,018%	49,982%
9953	535	2856328	540	2854245	1075	5710573	50,018%	49,982%
9954	558	2856886	535	2854780	1093	5711666	50,018%	49,982%
9955	530	2857416	528	2855308	1058	5712724	50,018%	49,982%
9956	552	2857968	511	2855819	1063	5713787	50,019%	49,981%
9957	528	2858496	573	2856392	1101	5714888	50,018%	49,982%
9958	542	2859038	541	2856933	1083	5715971	50,018%	49,982%
9959	516	2859554	563	2857496	1079	5717050	50,018%	49,982%
9960	516	2860070	542	2858038	1058	5718108	50,018%	49,982%
9961	507	2860577	534	2858572	1041	5719149	50,018%	49,982%
9962	559	2861136	552	2859124	1111	5720260	50,018%	49,982%
9963	526	2861662	530	2859654	1056	5721316	50,018%	49,982%
9964	534	2862196	512	2860166	1046	5722362	50,018%	49,982%
9965	540	2862736	540	2860706	1080	5723442	50,018%	49,982%
9966	513	2863249	539	2861245	1052	5724494	50,018%	49,982%
9967	563	2863812	562	2861807	1125	5725619	50,018%	49,982%
9968	551	2864363	544	2862351	1095	5726714	50,018%	49,982%
9969	522	2864885	561	2862912	1083	5727797	50,017%	49,983%
9970	552	2865437	531	2863443	1083	5728880	50,017%	49,983%
9971	520	2865957	546	2863989	1066	5729946	50,017%	49,983%
9972	535	2866492	514	2864503	1049	5730995	50,017%	49,983%
9973	532	2867024	560	2865063	1092	5732087	50,017%	49,983%
9974	552	2867576	571	2865634	1123	5733210	50,017%	49,983%
9975	551	2868127	533	2866167	1084	5734294	50,017%	49,983%
9976	531	2868658	538	2866705	1069	5735363	50,017%	49,983%
9977	529	2869187	547	2867252	1076	5736439	50,017%	49,983%
9978	536	2869723	558	2867810	1094	5737533	50,017%	49,983%
9979	552	2870275	548	2868358	1100	5738633	50,017%	49,983%
9980	518	2870793	575	2868933	1093	5739726	50,016%	49,984%
9981	579	2871372	589	2869522	1168	5740894	50,016%	49,984%
9982	541	2871913	534	2870056	1075	5741969	50,016%	49,984%
9983	545	2872458	534	2870590	1079	5743048	50,016%	49,984%
9984	539	2872997	525	2871115	1064	5744112	50,016%	49,984%
9985	536	2873533	526	2871641	1062	5745174	50,016%	49,984%
9986	530	2874063	565	2872206	1095	5746269	50,016%	49,984%
9987	551	2874614	522	2872728	1073	5747342	50,016%	49,984%
9988	520	2875134	533	2873261	1053	5748395	50,016%	49,984%
9989	541	2875675	559	2873820	1100	5749495	50,016%	49,984%
9990	532	2876207	517	2874337	1049	5750544	50,016%	49,984%
9991	555	2876762	543	2874880	1098	5751642	50,016%	49,984%
9992	533	2877295	553	2875433	1086	5752728	50,016%	49,984%
9993	532	2877827	546	2875979	1078	5753806	50,016%	49,984%
9994	561	2878388	546	2876525	1107	5754913	50,016%	49,984%
9995	559	2878947	531	2877056	1090	5756003	50,016%	49,984%
9996	552	2879499	537	2877593	1089	5757092	50,017%	49,983%
9997	535	2880034	559	2878152	1094	5758186	50,016%	49,984%
9998	536	2880570	551	2878703	1087	5759273	50,016%	49,984%
9999	556	2881126	549	2879252	1105	5760378	50,016%	49,984%
10000	544	2881670	533	2879785	1077	5761455	50,016%	49,984%

Figure 1. Comparison between rows from 9942 to 10000 of $TZ[x+2,x,x]$.

Axxxxxx The sum of the number of Primes between Squares and following Oblongs {1, 2, 3, 5, 6, 8, 9, 11, 13, 17, 19, 21, 24, 26, 30, 34, 35, 37, 40, 43, 47, 51, 53, 57, 61, 65, 69, 73, 77, 81, 86, 91, 97, 101, 106, 113, 116, 122, 128, 136, 141, 146, 153, 157, 163, 170, 176, 183, 189, 195, 200, 209, 216, 223, …}. The sum of the element in A089610.

Ayyyyyy The sum of the number of Primes between Oblongs and following Squares {1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 15, 18, 20, 22, 24, 27, 31, 35, 38, 42, 45, 48, 52, 57, 61, 64, 68, 73, 77, 81, 86, 90, 94, 99, 104, 106, 112, 118, 123, 127, 133, 137, 142, 149, 156, 159, 166, 174, 178, 183, 193, 200, 205, 211, …}. The sum of the element in A094189.

Azzzzzz The sum of the number of Primes between two consecutive Squares {2, 4, 6, 9, 11, 15, 18, 22, 25, 30, 34, 39, 44, 48, 54, 61, 66, 72, 78, 85, 92, 99, 105, 114, 122, 129, 137, 146, 154, 162, 172, 181, 191, 200, 210, 219, 228, 240, 251, 263, 274, 283, 295, 306, 319, 329, 342, 357, 367, 378, 393, 409, 421, 434, …}. The sum of the elements in A014085.

9.4 Number of Primes between consecutive Square numbers

A014085 Number of Primes between n^2 and $(n+1)^2$ {0, 2, 2, 2, 3, 2, 4, 3, 4, 3, 5, 4, 5, 5, 4, 6, 7, 5, 6, 6, 7, 7, 7, 6, 9, 8, 7, 8, 9, 8, 8, 10, 9, 10, 9, 10, 9, 9, 12, 11, 12, 11, 9, 12, 11, 13, 10, 13, 15, 10, 11, 15, 16, 12, 13, 11, 12, 17, 13, 16, 16, 13, 17, 15, 14, 16, 15, 15, 17, 13, 21, 15, 15, 17, 17, 18, 22, 14, 18, 23, 13,…}. This sequence represents the number of Primes in each row of $TZ[x+2,x,x]$. Suggested by Legendre's conjecture (still open) that for $n>0$ there is always a Prime between n^2 and $(n+1)^2$.

Conjecture to prove in this study: There are always over 2 Primes between two consecutive nonzero Squares.

For future studies:

- A060199 Number of Primes between n^3 and $(n+1)^3$. {0, 4, 5, 9, 12, 17, 21, 29, 32, 39, 49, 52, 58, 73, 76, 88, 92, 109, 117, 125, 140, 151, 159, 176, 188, 199, 207, 233, 247, 254, 267, 284, 305, 320, 346, 338, 373, 385, 416, 418, 437, 458, 481, 504, 517, 551, 555, 583, 599, 636, 648, 678, 686, 733, 723, 753, 810, …}. Conjecture: There are always over 3 (or 4?) Primes between two consecutive nonzero Cubes.
    - A216266 Number of Primes between n^3 and n^3+n (inclusive). {1, 0, 1, 1, 1, 0, 2, 0, 1, 1, 0, 1, 2, 2, 1, 2, 1, 3, 3, 3, 2, 4, 0, 3, 5, 4, 4, 2, 3, 2, 2, 5, 3, 3, 2, 5, 2, 3, 4, 5, 2, 3, 3, 5, 8, 5, 4, 5, 4, 3, 6, 6, 4, 4, 6, 5, 3, 7, 8, 2, 3, 6, 6, 5, 4, 5, 6, 5, 4, 4, 3, 4, 8, 8, 4, 5, 8, 7, 6, 5, 4, 5, 9, 6, 8, 8, 6, 8, 10, 6, 9, 11, …}. Conjecture: a(n)>0 for n>23.
- A061235 Number of Primes between n^4 and $(n+1)^4$. {0, 6, 16, 32, 60, 96, 147, 207, 283, 382, 486, 619, 773, 945, 1139, 1351, 1610, 1870, 2165, 2496, 2848, 3237, 3653, 4125, 4572, 5118, 5698, 6269, 6894, 7586, 8309, 9033, 9907, 10656, 11616, 12522, 13509, 14552, 15639, 16708, 18009, 19140, 20527, …}. Conjecture: There are always over 4 (or 6?) Primes between two consecutive nonzero power 4th.
- A062517 Number of Primes between n^5 and $(n+1)^5$. {0, 11, 42, 119, 273, 540, 954, 1573, 2456, 3624, 5181, 7177, 9666, 12797, 16514, 21098, 26454, 32836, 40134, 48760, 58508, 69714, 82277, 96723, 112702, 130639, 150488, 172617, 197039, 223915, 253318, 285540, 320450, 358839, 400159, 445011, 493504, …}. Conjecture: There are always over 5 (or 11?) Primes between two consecutive nonzero power 5th.
- And so on…

A007491 Smallest $Prime > n^2$ or $a(n)$ is the smallest Prime p such that $n^2 < p < n^2+n$ {2, 5, 11, 17, 29, 37, 53, 67, 83, 101, 127, 149, 173, 197, 227, 257, 293, 331, 367, 401, 443, 487, 541, 577, 631, 677, 733, 787, 853, 907, 967, 1031, 1091, 1163, 1229, 1297, 1373, 1447, 1523, 1601, 1693, 1777, 1861, 1949, 2027, 2129, 2213, 2309, 2411, 2503, …}.

A053001 Largest $Prime < n^2$. {3, 7, 13, 23, 31, 47, 61, 79, 97, 113, 139, 167, 193, 223, 251, 283, 317, 359, 397, 439, 479, 523, 571, 619, 673, 727, 773, 839, 887, 953, 1021, 1087, 1153, 1223, 1291, 1367, 1439, 1511, 1597, 1669, 1759, 1847, 1933, 2017, 2113, 2207, 2297, 2399, 2477, 2593, …}.

9.5 Number of Primes between consecutive Oblong numbers

The best trianz to see the number of Primes between consecutive Oblong numbers is $TZ[x + 1, x, x + 1]$.

-1	-10	-9	-8	7	-6	-5	-4	-3	-2	-1	0	1									
-2										2	3	4	5								
-3									6	7	8	9	10	11							
-4								12	13	14	15	16	17	18	19						
-5							20	21	22	23	24	25	26	27	28	29					
-6						30	31	32	33	34	35	36	37	38	39	40	41				
-7					42	43	44	45	46	47	48	49	50	51	52	53	54	55			
-8				56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71		
-9			72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	
-10		90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109

Figure 1. The trianz $TZ[x + 1, x, x + 1]$ has two central columns: the sequence A005563 (Square minus 1) numbers and the sequence A000290 Square numbers.

See that each line starts with an A002378 Oblong number and ends with the sequence A165900 Values of the Fibonacci polynomial $Y[y] = y^2 - y - 1$.

Let's consider the number of Primes between consecutive Oblong numbers from line -3 onwards the sequence A108309 {~~0, 2,~~ 2, 3, 2, 3, 3, 4, 4, 5, 3, 4, 6, 4, 6, 6, 4, 6, 7, 6, 8, 7, 5, 8, 9, 8, 7, 8, 9, 8, 9, 10, 10, 8, 10, 12, 5, 12, 12, 13, 9, 11, 11, 9, 13, 14, 9, 14, 14, 10, 10, 19, 14, 12, 12, 12, 12, 16, 15, 16, 15, 13, 18, 16, 16, 12, 16, 17, 15, 16, 18, 14, 15, 20, 18, 19, 14, 19, 20, 18, 16, …}. Except for the initial term, $a(n) \geq 2$ because in the interval $2n - 1$ of odd numbers there are always at least two Primes. For $n > 2$, this is the same as the number of Primes between $n^2 - n$ and $n^2 + n$, which is the sum of A089610 and A094189.

To check in Polyboctys the Oblong equivalence in Cubics, Quartics, Quintics, etc.

The sequence of the smallest Prime number greater than an Oblong number is A182047 {2, 3, 7, 13, 23, 31, 43, 59, 73, 97, 113, 137, 157, 191, 211, 241, 277, 307, 347, 383, 421, 463, 509, 557, 601, 653, 709, 757, 821, 877, 937, 997, 1061, 1123, 1193, 1277, 1361, 1409, 1483, 1567, 1657, 1723, 1811, 1901, 1987, 2081, 2179, 2267, 2357, 2459, 2551, …}.

The sequence of the largest Prime number smaller than an Oblong number is A161550 {2, 5, 11, 19, 29, 41, 53, 71, 89, 109, 131, 151, 181, 199, 239, 271, 293, 337, 379, 419, 461, 503, 547, 599, 647, 701, 751, 811, 863, 929, 991, 1051, 1117, 1187, 1259, 1327, 1399, 1481, 1559, 1637, 1721, 1801, 1889, 1979, 2069, 2161, 2251, 2351, 2447, 2549, 2647, …}.

9.6 The sequences of Primes

If we expand this paraboctys with $a = 1$ into more columns, we will find relatively close to the Oblong number sequence some Prime sequences like:

		OEIS				A186950							A028387							
		OEIS			A292509	A059425		A243138					A110331							
		OEIS		A212325	A332884	A126665		A126719	A275591		A108195	A082111	A165900	A002061	A014206	A027688	A027690	A048058	A260679	A005846
		a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		b	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
dif dif	dif	C	-283	-169	-109	-73	-49	-43	-19	-13	-7	-5	-1	1	2	3	5	11	17	41
		21	137	251	311	347	371	377	401	407	413	415	419	421	422	423	425	431	437	461
	40	20	97	211	271	307	331	337	361	367	373	375	379	381	382	383	385	391	397	421
2	38	19	59	173	233	269	293	299	323	329	335	337	341	343	344	345	347	353	359	383
2	36	18	23	137	197	233	257	263	287	293	299	301	305	307	308	309	311	317	323	347
2	34	17	-11	103	163	199	223	229	253	259	265	267	271	273	274	275	277	283	289	313
2	32	16	-43	71	131	167	191	197	221	227	233	235	239	241	242	243	245	251	257	281
2	30	15	-73	41	101	137	161	167	191	197	203	205	209	211	212	213	215	221	227	251
2	28	14	-101	13	73	109	133	139	163	169	175	177	181	183	184	185	187	193	199	223
2	26	13	-127	-13	47	83	107	113	137	143	149	151	155	157	158	159	161	167	173	197
2	24	12	-151	-37	23	59	83	89	113	119	125	127	131	133	134	135	137	143	149	173
2	22	11	-173	-59	1	37	61	67	91	97	103	105	109	111	112	113	115	121	127	151
2	20	10	-193	-79	-19	17	41	47	71	77	83	85	89	91	92	93	95	101	107	131
2	18	9	-211	-97	-37	-1	23	29	53	59	65	67	71	73	74	75	77	83	89	113
2	16	8	-227	-113	-53	-17	7	13	37	43	49	51	55	57	58	59	61	67	73	97
2	14	7	-241	-127	-67	-31	-7	-1	23	29	35	37	41	43	44	45	47	53	59	83
2	12	6	-253	-139	-79	-43	-19	-13	11	17	23	25	29	31	32	33	35	41	47	71
2	10	5	-263	-149	-89	-53	-29	-23	1	7	13	15	19	21	22	23	25	31	37	61
2	8	4	-271	-157	-97	-61	-37	-31	-7	-1	5	7	11	13	14	15	17	23	29	53
2	6	3	-277	-163	-103	-67	-43	-37	-13	-7	-1	1	5	7	8	9	11	17	23	47
2	4	2	-281	-167	-107	-71	-47	-41	-17	-11	-5	-3	1	3	4	5	7	13	19	43
2	2	1	-283	-169	-109	-73	-49	-43	-19	-13	-7	-5	-1	1	2	3	5	11	17	41
2	0	0	-283	-169	-109	-73	-49	-43	-19	-13	-7	-5	-1	1	2	3	5	11	17	41
2	-2	-1	-281	-167	-107	-71	-47	-41	-17	-11	-5	-3	1	3	4	5	7	13	19	43
2	-4	-2	-277	-163	-103	-67	-43	-37	-13	-7	-1	1	5	7	8	9	11	17	23	47
2	-6	-3	-271	-157	-97	-61	-37	-31	-7	-1	5	7	11	13	14	15	17	23	29	53
2	-8	-4	-263	-149	-89	-53	-29	-23	1	7	13	15	19	21	22	23	25	31	37	61
2	-10	-5	-253	-139	-79	-43	-19	-13	11	17	23	25	29	31	32	33	35	41	47	71
2	-12	-6	-241	-127	-67	-31	-7	-1	23	29	35	37	41	43	44	45	47	53	59	83
2	-14	-7	-227	-113	-53	-17	7	13	37	43	49	51	55	57	58	59	61	67	73	97
2	-16	-8	-211	-97	-37	-1	23	29	53	59	65	67	71	73	74	75	77	83	89	113
2	-18	-9	-193	-79	-19	17	41	47	71	77	83	85	89	91	92	93	95	101	107	131
2	-20	-10	-173	-59	1	37	61	67	91	97	103	105	109	111	112	113	115	121	127	151
2	-22	-11	-151	-37	23	59	83	89	113	119	125	127	131	133	134	135	137	143	149	173
2	-24	-12	-127	-13	47	83	107	113	137	143	149	151	155	157	158	159	161	167	173	197
2	-26	-13	-101	13	73	109	133	139	163	169	175	177	181	183	184	185	187	193	199	223
2	-28	-14	-73	41	101	137	161	167	191	197	203	205	209	211	212	213	215	221	227	251
2	-30	-15	-43	71	131	167	191	197	221	227	233	235	239	241	242	243	245	251	257	281
2	-32	-16	-11	103	163	199	223	229	253	259	265	267	271	273	274	275	277	283	289	313
2	-34	-17	23	137	197	233	257	263	287	293	299	301	305	307	308	309	311	317	323	347
2	-36	-18	59	173	233	269	293	299	323	329	335	337	341	343	344	345	347	353	359	383
2	-38	-19	97	211	271	307	331	337	361	367	373	375	379	381	382	383	385	391	397	421
2	-40	-20	137	251	311	347	371	377	401	407	413	415	419	421	422	423	425	431	437	461

Particularly, when coefficient c is a Prime, we can see A014556 Euler's "Lucky" numbers: n such that m^2-m+n is Prime for m=0..n-1.

Acknowledgments

I would like to thank all the essential support and inspiration provided by Mr. H. Bli Shem and my Family.

References

[1]*The On-Line Encyclopedia of Integer Sequences*, available online at http://oeis.org.

[2]